Universality of turbulence and engineering simulations.

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April 11, 2014

S. Orszag (Princeton), H. Chen, J. Wanderer, R. Shock, I. Staroselsky, (EXA Corp.); J. Schumacher (Ilmenau, Germany) A. Polyakov, A. Smits , S. Bailey, M.Hultmark (Princeton), Diego Donzis (Texas A&M), Katepalli Sreenivasan (NYU), J.D. Scheel ,

1. S.A. Orszag, Phys.Fluids 12 250 (1969); S.A. Orszag and G.S. Patterson, "Numerical Simulation of Three-Dimensional Homogeneous Isotropic Turbulence", Phys.Rev.Lett.28, 76 (1972). Channel flow; Chebyshev etc.

2. J.W. Deardorff, J. Fluid Mech. 41,453 (1970); 7, 120 (1971);

3. B. Launder, D.B. Spalding, "The numerical computations of turbulent flows", Comp.Methods.Mech.Eng. 3, 269 (1974)

Orszag Patterson: DNS, HIT, 64^3, spectrum ...

$$N = O(Re^{\frac{9}{4}}); \qquad W = O(Re^3)$$

$$W = O(Re^4)$$

Deardorff, Channel flow; Atmospheric Boundary layer; Smagorinsky model+mixing length; still, no log-layer...

Launder and Spalding: $\mathcal{K} - \epsilon$-model; started the field of commercial CFD. Intellectually an extremely interesting and important development. Led to time-dependent simulations: VLES, PANs etc.

Since: Mesh size: DNS: HIT: $m = \frac{4096^3}{64^3} \approx 3 \times 10^5$; $w \approx 16 \times 10^6$ *(due to intermittency, it may be not enough for the full DNS); Convection:* $Ra \approx 10^{10}$; *Channel/pipes,BL ..*

DNS - a remarkably powerful scientific tool, *if one asks question first.*

$\mathcal{K} - \epsilon$ **model (VLES) became an indispensable part of an engineering design cycle; Total annual sale of commercial CFD codes (structures excluded) is:**

$$s \approx 5 \times 10^8 USD$$

and rapidly grows. Only $3 - 5\%$ **of customers use LES. (F. Boysan, Fluent President; H.Chen, EXA, Senior VP)**

Industry standards.

1. Accuracy on all cars: C_d: $\approx 2-4\%$.

2. All tests are blind.

3. Heat transfer: Nu must be calculated with $\approx 4-5\%$.

3. The model $= const$.

4. Tiny details of the system are important: logos, pillars, tires....

LES:

If size of computational mesh is Δ, filter out all fluctuations on the scales $r \leq \Delta$ and compute the remaining field. The goal was to fix Δ and achieve scaling of computational work

$$W = O(Re^0)$$

The main question is: how do you write the remaining equation for "resolved" scales $\mathbf{u}^< \equiv \mathbf{u}$? According to Kolmogorov's theory : if Δ is in IR:

$$const = \overline{\mathcal{E}} = -\frac{5}{4}\frac{\overline{(\delta_\Delta u)^3}}{\Delta} = \nu(\Delta)\overline{S_{ij}^2}$$

Applied locally, this relation becomes

$$\nu(\Delta) = |u(x+\Delta)-u(x)|\Delta = \frac{|u(x+\Delta)-u(x)|}{\Delta}\Delta^2 \approx |S|\Delta^2$$

Smagorynsky model.

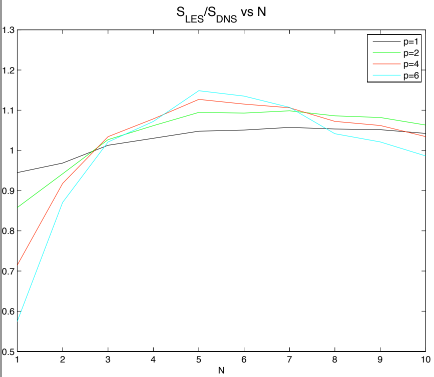
Dynamic, variational, constraint, models for Δ

....

State - of-the-art-LES. Oberai, Wanderer. DNS (256^3) vs LES (32^3).

$$s_n(r) = (\overline{(u(x+r)-u(x))^n}^{\frac{1}{n}}_{LES} / (\overline{(u(x+r)-u(x))^n}^{\frac{1}{n}}_{DNS}.$$

Root-n mean.



In this talk I will present a mathematical tool leading to all these models as different limiting cases. I will be able to produce estimates of accuracy of different models and assess their performance on a few examples.

Let us start with the smallest scales.

$$\frac{\partial u}{\partial x} = \lim_{\Delta \to 0} \frac{u(\mathbf{x} + \Delta \mathbf{i}) - u(\mathbf{x})}{\Delta}$$

$$\delta_r u = u(x+r) - u(x)$$

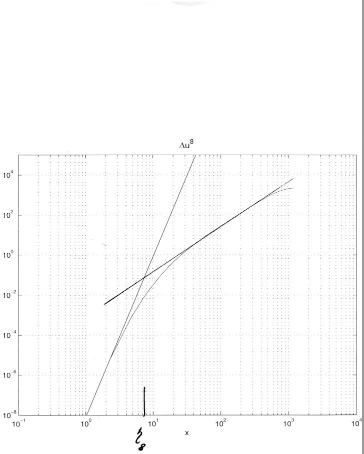
$r \approx L;$ $L >> r >> \eta;$ $\eta >> r$

$LSF;$ IR $\delta_r u \approx \frac{\partial u}{\partial x} r$

$$S_n(r) = \overline{(\delta_r u)^n}$$

$$S_n(r)=\overline{(\delta_r u)^n}=C_K(\mathcal{E}L)^{\frac{n}{3}}(\frac{r}{L})^{\xi_n};\quad S_n\propto\overline{(\frac{\partial u}{\partial x})^n}r^n$$

Definition of the dissipation scale η_n $(n=8)$. KRS



$$S_{2n}(\eta_{2n}) = \overline{(\frac{\partial u}{\partial x})^{2n}}\eta_{2n}^{2n} = A_{2n}\eta_{2n}^{\xi_{2n}}$$

Dissipation scale is a crossover from analytic to singular intervals of structure functions.

$$\overline{(\frac{\partial u}{\partial x})^{n}} \propto Re^{\rho_n} = (\frac{u_{rms}L}{\nu})^{\rho_n}$$

$$S_{2n}(\eta_{2n})/\eta_{2n} \approx S_{2n+1}(\eta_{2n+1})/\nu$$

DISSIPATION SCALE η_n DEPENDS UPON MOMENT ORDER n.

$$\eta_n \approx LRe^{\frac{1}{\xi_n - \xi_{n+1} - 1}}$$

For the full DNS including small-scale effects: $W = O(Re^4)$.

Dissipation scale is a fluctuating parameter:

$$Re_\eta = \frac{\eta \delta_\eta u}{\nu} \approx 1$$

PDF $Q(\eta)$. Universal. (HIT, Pipes, convection.)

FROM NS EQUATIONS:

Moments of derivatives Exponents.

$$\rho_n = n + \frac{\xi_{2n}}{\xi_{2n} - \xi_{2n+1} - 1}$$

$$d_n = n + \frac{\xi_{4n}}{\xi_{4n} - \xi_{4n+1} - 1}$$

$$\alpha_n = n + \frac{\xi_{3n}}{\xi_{n} - \xi_{3n+1} - 1}$$

$$\xi_n = 0.383n/(1 + n/20)$$

Simulations. Velocity derivatives

J. SHUMACHER, K.R. SREENIVASAN AND VY (2007).

NUMERICS 1024^3**; ISOTROPIC TURBULENCE;**

$$4 \leq R_\lambda = \sqrt{\frac{5}{3}\frac{1}{\mathcal{E}\nu}}u_{rms}^2 \leq 123$$

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \nu_0 \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\mathbf{f} = \mathcal{P} \frac{\mathbf{u}(\mathbf{k}, t)}{\sum_{k_f} |\mathbf{u}(\mathbf{k}, \mathbf{t})|^2} \delta_{\mathbf{k}, \mathbf{k}_{\mathbf{k_f}}}$$

$$\mathbf{k}_f = (1,1,2);\ (1,2,2);\ \mathcal{E} = \overline{\nu(\frac{\partial u_i}{\partial x_j})^2} = \mathcal{P} = cons$$

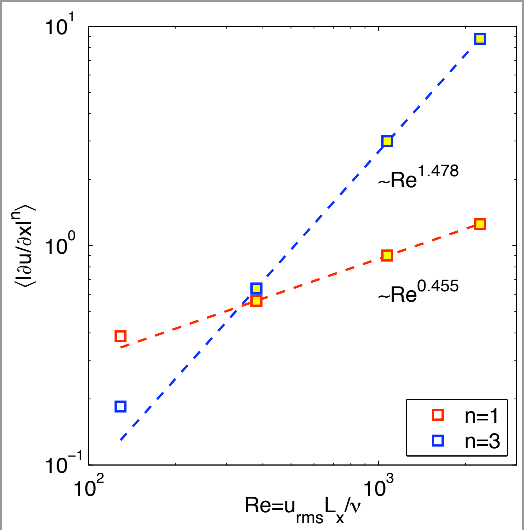
***Re* is varied by variation of viscosity.**

$$\frac{\partial u}{\partial x} \approx \frac{\delta_\eta u}{\eta} \equiv \frac{u(x+\eta) - u(x)}{\eta} = \frac{(u(x+\eta) - u(x))^2}{\nu}$$

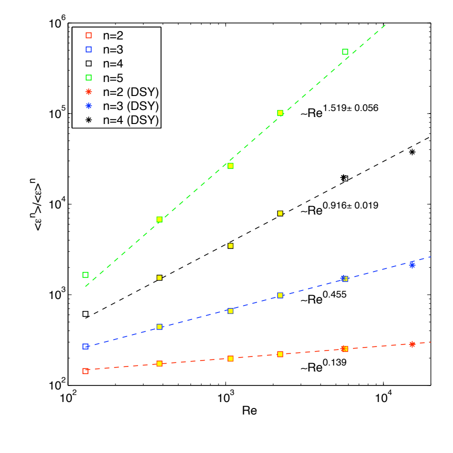
η is a displacement in analytic interval. This establishes relations between SFs in the IR range and derivatives.

$$\overline{(\frac{\partial u}{\partial x})^n} = \overline{(\frac{(\delta_r u)^2}{\nu})^2)^n} \propto Re^n S_{2n}(\eta)$$

MOMENTS OF VELOCITY DERIVATIVES.



anomalous scaling of dissipation rate. Schumacher, KRS, VY, Donzis,



Universality; Schumacher et al. 2014

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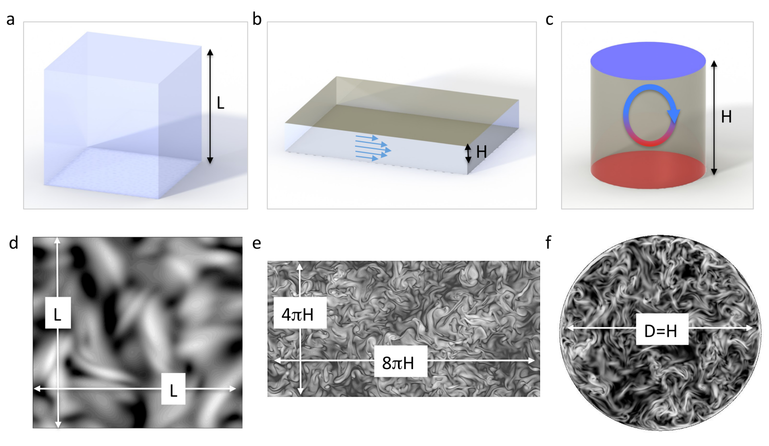

Universality; Schumacher et al. 2014

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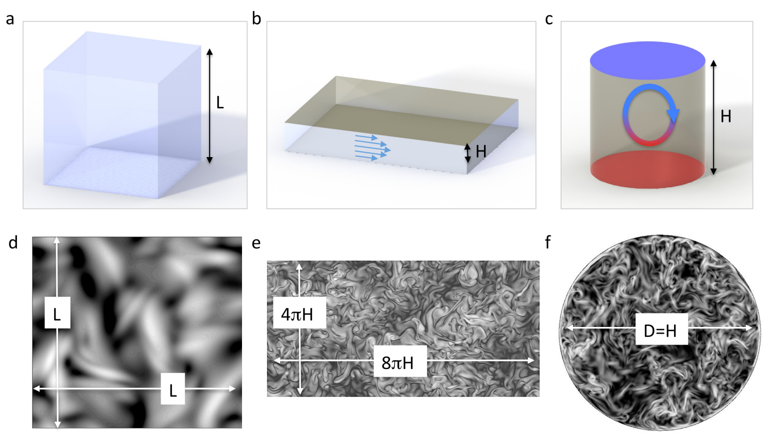
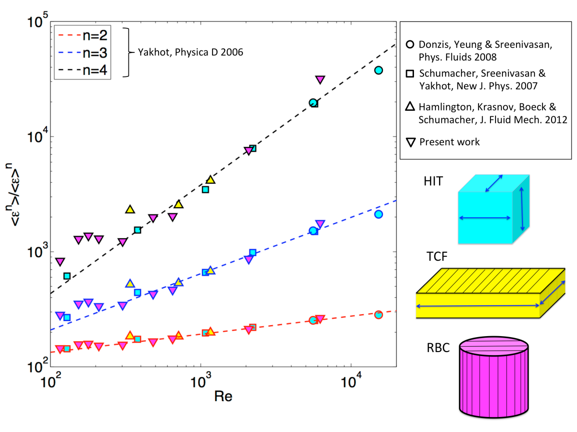
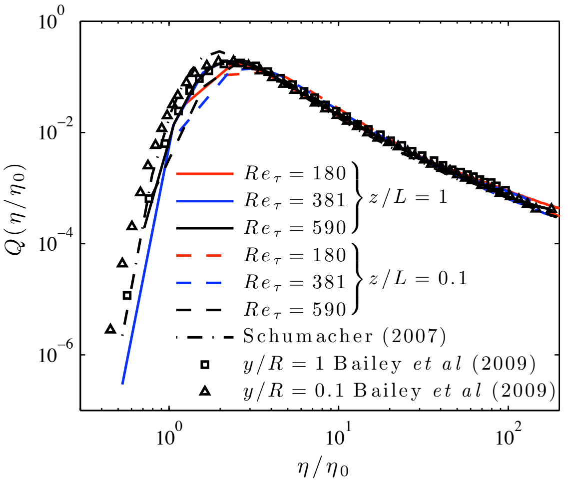


Universality of derivatives.(Schumacher, Sheele, Donzis, Sreenivasan, Krasnov, VY. 2014




10^0
10^{-2}
10^{-4}
10^{-6}
$Q(\eta/\eta_0)$
10^0
10^1
10^2
η/η_0
$Re_\tau = 180$
$Re_\tau = 381$
$Re_\tau = 590$
$z/L = 1$
$Re_\tau = 180$
$Re_\tau = 381$
$Re_\tau = 590$
$z/L = 0.1$
Schumacher (2007)
$y/R = 1$ Bailey *et al* (2009)
$y/R = 0.1$ Bailey *et al* (2009)

TRANSITION. GAUSSIAN POINT.

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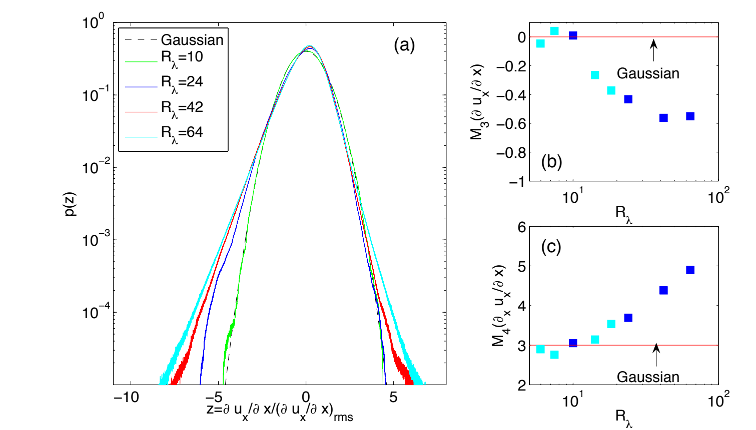
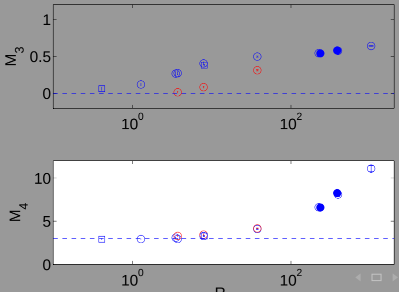
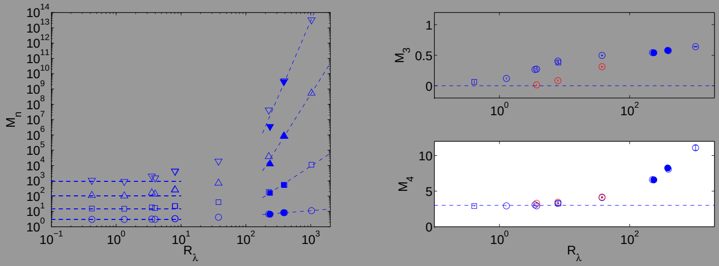


TRANSITION. GAUSSIAN POINT. $0.5 \leq R_\lambda \leq 200$. Diego Donzis. (2013).



HIT Driven by RF. Diego Donzis.



At $R_\lambda < 10$ the flow is a dynamical system described by a few modes. Quasicoherent (mixed) state.

at

$$R_\lambda \approx 9.0 \;-\; 10.$$

TRANSITION WAS SMOOTH (NO JUMPS.)

SUMMARY: Coming from low Reynolds numbers, we found a transition in VELOCITY DERIVATIVES: at the GAUSSIAN transition point to FULLY DEVELOPED TURBULENCE

$$R_\lambda^{tr} = \sqrt{\frac{5}{3}\frac{1}{\nu_{tr}\mathcal{E}}}u_{rms}^2 \approx 9.0 \; - \; 10$$

Please, remember this number!

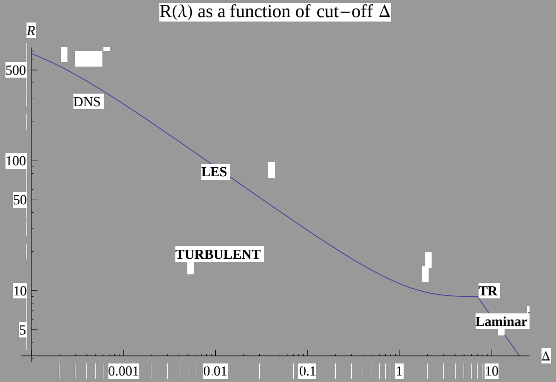


Figure: PROGRAM: study variation of the Reynolds number $R_{\lambda}(\Delta)$ with the u.v. cut-off (filtering scale) Δ in a turbulent flow with $R_{\lambda,0} \approx 1000$ and the integral scale $L = 2\pi/\Lambda_f \approx 10$. By successive small-scale filtering we will derive formal expressions for DNS,LES and VLES.

At the transition point $Re_{tr} = u_0/(\nu_{tr}\Lambda_f) \approx 9 - 10$:

$$\frac{D\mathbf{u_0}}{Dt} = -\nabla p + \nu_{tr}\nabla^2\mathbf{u_0} + \mathbf{F}(\Lambda_f)$$

$$\nu \ll \nu_{tr}; \qquad \mathbf{u} = \mathbf{u_0} + \mathbf{v}$$

$$\frac{D\mathbf{v}}{Dt} = -\nabla p - +\nu\nabla^2\mathbf{v} + \mathbf{f}$$

$$f = -\mathbf{u_0}\cdot\mathbf{v} - \mathbf{v}\cdot\mathbf{u_0} + (\nu - \nu_{tr})\nabla^2\mathbf{u_0}$$

The model.

Then:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nu_0 \nabla^2 \mathbf{v} + \mathbf{f}$$

$$\mathbf{f} = -\nabla_i (u_{0,i} \mathbf{v} + v_i \mathbf{u}_0)$$

$\mathbf{v}$ is excited by interaction with $\mathbf{u}_0$ and $0 < \nu_0 \leq \nu_{tr}$.

$$\overline{\mathbf{f}} = 0$$

$$\overline{f_i(\mathbf{k},\omega)f_j(\mathbf{k}',\omega')} = 2D_0 P_{ij}(\mathbf{k})\mathbf{k}^{-\mathbf{y}}\delta(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega'$$

$$\frac{1}{L} = \Lambda_f \le k \le \Lambda_0$$

$$f(k < \Lambda_f) = 0; \qquad F(k \ge \Lambda_f) = 0$$

$$U = \sqrt{D_0/(\nu_0\Lambda_0^2)}; \qquad X = 1/\Lambda_0; \; D_0 \propto \mathcal{E}$$

$$\frac{\partial \mathbf{u}}{\partial T} + \hat{\lambda}_0 \mathbf{u} \cdot \nabla \mathbf{u} = -\hat{\lambda}_0 \nabla p + \nabla^2 \mathbf{u} + \frac{\mathbf{f}}{\sqrt{D_0 \nu_0 \Lambda_0^2}}$$

“bare” Reynolds number

$$\hat{\lambda}_0^{\;2} = \frac{D_0}{\nu_0^3 \Lambda_0^{\epsilon}}$$

$\mathbf{u} \equiv \mathbf{v}.$

We fix $\Lambda_f = const$, $D_0 \propto \mathcal{E} = \mathcal{P}$ and set $\nu_0 \to 0$ so that $\hat{\lambda}_0 \to \infty$ and $\Lambda_0 \to \infty$. The model mimics velocity fluctuations at $r < L = \Lambda_f$ caused by instability of the large-scale flow, forcing, etc. The secondary effects like eddy noise are consequences of f. Now we derive large-scale equations at the scales $1/\Lambda_0 \leq r \leq 1/\Lambda_f$

RNG. FNS (1976), Martin, DeDomonisis (1978); (Amplitudes): Orszag, VY. (1986); Smith, VY (1992); VY, Speziale et al. (1992). LARGE REYNOLDS NUMBER.

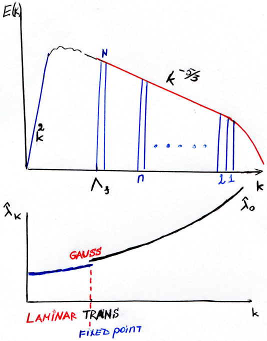


Figure: Schematic representation of scale elimination procedure and variation of dimensionless coupling constant $\hat{\lambda}(k)$. $Re = Re(k)$.

Eliminating modes from the interval leads to formally exact $SGM(\Delta)$:

$$2\pi/\Delta = \Lambda_0^{-r} \leq k \leq \Lambda_0 = 1/\eta_K$$

$$\frac{\partial \mathbf{u}^<}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}^< =$$

$$-\nabla p + (\nu_0 + \Delta\nu)\nabla^2 \mathbf{u}^< + \mathbf{F} + \mathbf{f} + \boldsymbol{\Delta}\mathbf{f} + HOT$$

$$\Delta\nu = A_d\frac{D_0}{\nu_0^2}[\frac{e^{\epsilon r}-1}{\epsilon\Lambda_0^{\epsilon}}+O(\frac{k^2}{\Lambda_0^{\epsilon+2}}\frac{e^{(\epsilon+2)r}-1}{\epsilon+2})+O(\hat{\lambda}_0^4$$

$\epsilon = 4+y-d$ **and**

$A_d = \hat{A}_d\frac{S_d}{(2\pi)^d};\ \hat{A}_d = \frac{1}{2}\frac{d^2-d}{d(d+2)}$.

$$\frac{2\pi}{\eta} = \Lambda_0 \rightarrow \Lambda(r) = \Lambda_0 e^{-r} = 2\pi/\Delta$$

$$\frac{e^r-1}{\Lambda_0} = \frac{\Delta-\Delta_0}{2\pi}$$

Due to Galileo invariance, high-order $(n > 1)$ terms (HOT) generated by scale-elimination are of the order:

$$HOT = [\sum_{n=2}^{\infty} \hat{\lambda}_1^{\,2n} \tau_0^{n-1} (\partial_t \mathbf{u}^< + \mathbf{u}^< \cdot \nabla)^{\mathbf{n}}]\mathbf{u}^< + O(\hat{\lambda}_0^4 \nabla S_{ij}^2 \frac{1}{\Lambda_0^2} \frac{e^{(\epsilon+2)r} - 1}{\epsilon + 2}) + \cdots$$

with $\tau_0 \approx 1/(\nu_0 \Lambda_0^2)$ **and** $\hat{\lambda}_1 = \hat{\lambda}_0 (e^{\epsilon r} - 1)$.

Eliminating modes from the next shell (doubling the "filtering scale") and the next one → *LES*:

$$\Lambda_0^{-2r} \leq k \leq \Lambda_0 e^{-r}$$

$$\frac{\partial \mathbf{u}^<}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}^< =$$

$$-\nabla p + (\nu_0 + \Delta\nu)\nabla^2 \mathbf{u}^< + \mathbf{F} + \mathbf{f_1} + \mathbf{\Delta f_1} + HOT_1$$

"Real life charm". Sub-grid model for Reynolds stress as a function of Δ up to second order.

$$\sigma_{ij}^{(2)} = \hat{\lambda}^2 \nu(\Delta) S_{ij} - \hat{\lambda}_1^4(\Delta)\nu(\Delta)\frac{D}{Dt}[\tau(\Delta)(S_{ij}+S_{ji}] -$$

$$\hat{\lambda}_1^4(\Delta)\nu(\Delta)\beta_2\left(\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}+\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}\right)+\beta_3\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}]$$

+ burnett's terms etc. No way one can write down higher orders.

Origin of Smagorinsky model. Keep only first term:

$$\sigma^1_{ij} \approx \nu(\Delta) S_{ij} \approx \frac{\mathcal{E}\Delta^4}{\nu^3} \nu(\Delta) S_{ij}$$

$$\nu(\Delta) = |u(x+\Delta) - u(x)|\Delta \approx |S|\Delta^2$$

Trouble. Eliminating shells from the interval $\pi/\Delta \le k \le \Lambda_0$ gives an estimate:

$$\nu(\Delta) = \frac{\nu(\Delta)S^2\Delta^4}{\nu(\Delta)^2}\sum_{n=0}^{\infty}\hat{\lambda}^n(\Delta)\alpha_n(\frac{\nu(\Delta)S^2\Delta^4}{\nu^3(\Delta)})^n +$$

$$\sum_{n=2}^{\infty}\lambda_1^{2n}(k\Delta)^{2n}$$

In the zeroth order - Smagorinsky

$$\nu(\Delta) \approx |S|\Delta^2$$

First term gives Smagorinsky. Are remaining ones large or small ?

How do coefficients λ_i vary with Δ?

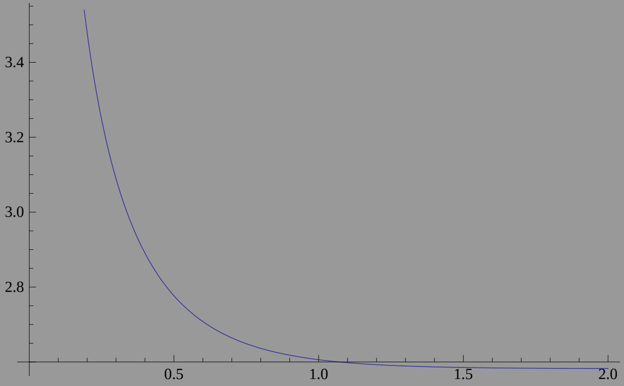


Figure: Dimensionless coupling constant $\hat{\lambda}(r)$ as a function of the length-scale $\Lambda(r)$. $\hat{\lambda}_0 = 1000$. For $\Delta \to 2\pi/\Lambda_f = L$, the dimensionless $\hat{\lambda}(\Delta) \to 2.58$

$$\hat{\lambda}_1(r) = \frac{\sqrt{\epsilon}\left(e^{\frac{\epsilon r}{2}} - 1\right)}{\sqrt{\frac{\epsilon}{\hat{\lambda}_1^2(0)} + 3A_d\left(e^{\frac{\epsilon r}{2}} - 1\right)}} \tag{1}$$

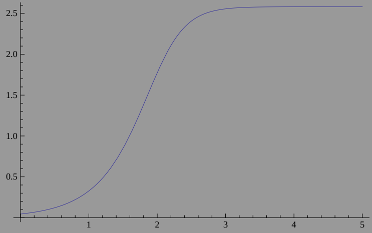


Figure: $\hat{\lambda}_1(r)$ as a function of the length-scale r grows with filtering. HOT are not small ! (LES!!!)

Intermediate summary:

1. $SGE = NS + \nu(\Delta) + HOT$.

2. As $\Delta \to 0$**,** $\nu(\Delta) \to \nu_0$**,** $HOT \to 0$**. (DNS)**

3. For Δ **in IR, no small parameter. Nothing can be neglected. (LES)**

4 . The effective viscosity is recovered at the large scales only ($k\Delta << 1$**).**

5. When $\Delta \to \Lambda_f$**,** $\hat{\lambda}(\Delta) \to 2.58$**.**

Let us investigate this limit in some details.

Fixed- point Reynolds number. Lowest - order ϵ expansion. (VY,)Orszag, VY. Smith...

$$2D_oS_d/(2\pi)^d = 1.59\mathcal{E}$$

$$C_K \approx 1.61$$

$$\nu_{\Lambda_f} = 0.084\frac{\mathcal{K}^2}{\mathcal{E}}$$

$$\mathcal{K}(t) \propto t^{-\gamma}; \qquad \gamma \approx 1.47$$

$$10\nu_T\Lambda_f^2 = \mathcal{K} = u_{rms}^2/2$$

$$\nu(\Lambda_f) = (\frac{3}{2}\hat{A}_d \times 1.594)^{\frac{1}{3}}(\frac{\mathcal{E}}{\Lambda^4})^{\frac{1}{3}} = 1.15(\frac{\mathcal{E}}{\Lambda^4})^{\frac{1}{3}}$$

$$\hat{\lambda}^* = \sqrt{\frac{D_0 S_d/(2\pi)^d}{\nu_T^3 \Lambda_f^4}} =$$

$$\frac{\sqrt{0.8 \times 400 \mathcal{E} \nu_T}}{u_{rms}^2}$$

$$\frac{\sqrt{0.8 \times 400 \times \frac{5}{3}}}{R_\lambda^{fp}} = \sqrt{\frac{4}{3\hat{A}_d}} = 2.58$$

Thus, the coupling constant $\hat{\lambda}^* = 2.58$ **obtained from the lowest order of the** ϵ**-expansion:**

$$R_\lambda^{fp} \approx 9.0$$

Therefore, $Re^{fp} = R_\lambda^{tr} \approx 9.0 - 10.$

IF TRANSITION IS SMOOTH AT FP

$$\frac{D\mathbf{u}^{tr}}{Dt} + \nabla p - \nu^{tr}\nabla^2\mathbf{u}^{tr}$$
$$\approx \frac{D\mathbf{u}^{fp}}{Dt} + \nabla p - \nu^{fp}\nabla^2\mathbf{u}^{fp} + HOT$$

$$\nu^{tr} \approx \nu^{fp}; \qquad \mathbf{u}^{tr} = \mathbf{u}^{fp} = \mathbf{u_0}$$

$$HOT = 0$$

At the scales $r > L = 2\pi/\Lambda_f$ the dynamics of a turbulent flow are described by the NS equations with $\nu = \nu^{fp}$. No high-order non-linearities etc.

This is the domain of RANS or VLES dominating engineering modeling.

Landau theory of transition to turbulence

1. **Laminar (coherent quasi-steady flow $\mathbf{u_o}$.**
2. **Perturbation: $\mathbf{u} = \mathbf{u_o} + \mathbf{v_1}(\mathbf{x}, \mathbf{t})$.**
3. **First unstable mode: $\mathbf{v_1} = A(t)f(\mathbf{r})$.**

$$\frac{d|A|^2}{dt} = 2\gamma_1|A|^2 - \alpha|A|^4$$

$$\gamma_1 = c(Re - Re_{tr}); \;\; \alpha > 0$$

$$A_{max} \propto \sqrt{Re - Re_{tr}}$$

Landau's theory of transition to turbulence.

$$\mathbf{u} = \mathbf{u_0} + \mathbf{u}_1 \equiv \mathbf{u}^{\mathbf{tr}} + \mathbf{u_1}$$

$$\mathbf{u}_{0,t} + \mathbf{u_0} \cdot \nabla \mathbf{u_0} = -\nabla p + \nu^{fp} \nabla^2 \mathbf{u}_0 + \mathbf{F}$$

$$\frac{\partial \mathbf{u}_1}{\partial t} + \mathbf{u}_0 \cdot \nabla \mathbf{u}_1 + \mathbf{u_1} \cdot \nabla \mathbf{u}_0 =$$

$$-\nabla p_1 + \nu^{fp} \nabla^2 \mathbf{u}_1 + \psi + HOT$$

If $\mathbf{u}_1 \propto Ae^{i\omega}$, then according to Landau's theory:
and

$$u_1 \propto A_{max} \propto \sqrt{Re - Re_{tr}}$$

$$HOT \approx \mathbf{u}_0 \cdot \nabla \mathbf{u}_1 \approx u_0^2 \Lambda_f \sqrt{Re - Re_{tr}}$$

This is the estimate of accuracy of VLES (RANS) modeling.

1. In the IR the coarse-grained (LES) equations are strongly nonlinear and require a lot of thinking (resumption of the series...). LES are a bit problematic.

2. Second problem is BC which is also to be dealt with nonperturbatively. One can use dynamic approach. Early attempts (VY, Bailey, Smits, JFM).

Engineering simulations.

1. To be useful for design the model must be able to predict flow features not “postdict”.
2. Therefore, the model and all coefficients must be fixed and not vary from flow to flow.
3. It has to be fast: a couple of days max per calculation.
4. Universal.

POWERFLOW.

EXA CORPORATION.

H. Chen, I. Staroselsky, R. Shock, J. Wanderer, O. Filippova, R. Zhang, J. Sacco.

LBG$\mathcal{KE}$ Model.

$$\partial_t f + \mathbf{v} \cdot \nabla f = -\frac{f - f^{eq}}{\tau}$$

$$\tau_{hit} = \frac{3}{2} \times 0.0845 \mathcal{K}/\mathcal{E} \rightarrow \nu_{turb} = \frac{2}{3} \mathcal{K} \tau_{turb}$$

$$\tau = \tau_0 + \Psi(\mathcal{K}/\mathcal{E}, S^{-1}, G)$$

$$\tau = \tau_0 + 0.0845 \frac{\mathcal{K}}{\mathcal{E}\sqrt{1 + \gamma\eta^2}}; \qquad \eta = \mathcal{K}S/\mathcal{E}$$

$$\eta \to 0; \nu_T \to 0.0845\mathcal{K}^2/\mathcal{E}$$

$$\eta \to \infty; \;\; \nu_T \propto \frac{\mathcal{K}}{S} \to 0$$

Reynolds stress: in the second order of CE expansion:

$$\sigma_{i,j}^{(2)} = \nu_{turb} S_{ij} +$$

$$\nu_{turb} \frac{D}{Dt}(\nu_{turb} S_{ij}) -$$

$$-\frac{\mathcal{K}^3}{\mathcal{E}^2}[C_1 \frac{\partial u_i}{\partial x_\alpha}\frac{\partial u_j}{\partial x_\alpha} + C_2(\frac{\partial u_i}{\partial x_\alpha}\frac{\partial u_\alpha}{\partial x_j} + \frac{\partial u_j}{\partial x_\alpha}\frac{\partial u_\alpha}{\partial x_j}) +$$

$$C_3 \frac{\partial u_\alpha}{\partial x_i}\frac{\partial u_\alpha}{\partial x_j}] + allorders$$

The BGK equation contains all possible non-linear models.

$$\frac{D\mathcal{K}}{Dt} = \nu_T S_{ij}^2 - \mathcal{E} + 1.39\nabla(\nu_T \nabla \mathcal{K})$$

$$\frac{D\mathcal{E}}{Dt} = 1.42\nu_T S_{ij}^2 \frac{\mathcal{E}}{\mathcal{K}} - 1.68\frac{\mathcal{E}^2}{\mathcal{K}} + \mathcal{R} + 1.39\nabla(\nu_T \nabla \mathcal{E})$$

VY-Smith:

$$\mathcal{R} = 2\nu_0 \overline{\frac{\partial u_i}{\partial x_l}\frac{\partial u_j}{\partial x_l}} S_{ij} \approx -\frac{\nu_T S^3 (1-\eta/4.38)}{1+\gamma\eta^3}$$

$$S \to 0; \;\; \mathcal{R} \to 0$$

$$\eta \to \infty; \;\; \mathcal{R} \propto +\mathcal{E}^2/\mathcal{K}$$

The model is fixed !!!

Turbulent flow past 3D circular cylinder; $Re = 2 \times 10^6$.

C. Bartlett et al.

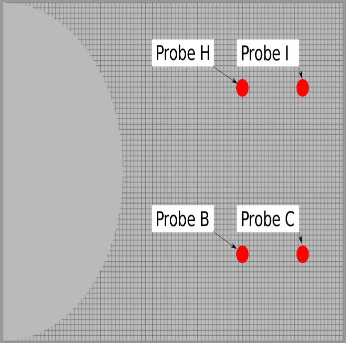


Figure: a. Probe layout in the flow past cylinder

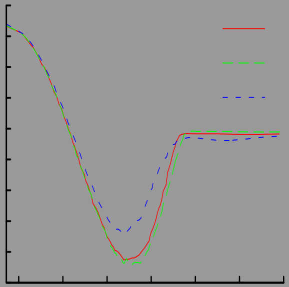


Figure: Pressure surface coefficients C_p for different resolutions. Left $Re = 10^5$. Right: 2×10^6.

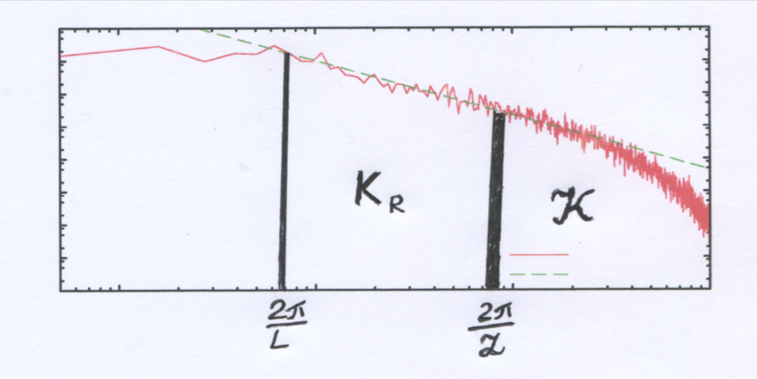


Figure: Length and energy scales in a flow. Dotted line: $E(K) = C_{\mathcal{K}} \mathcal{E}^{\frac{2}{3}} k^{-\frac{5}{3}}$. $C_{\mathcal{K}} \approx 1.5 - 1.8$.

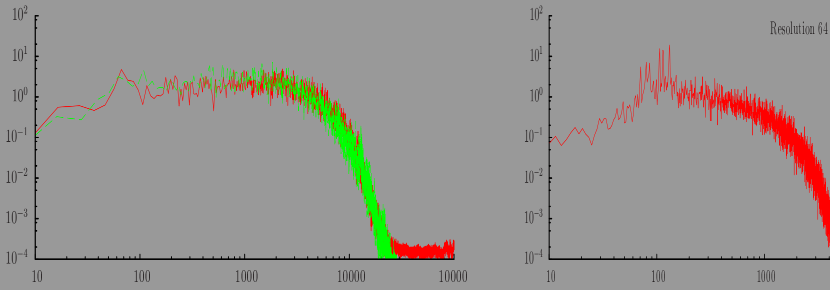


Figure: Compensated energy spectra $E(k)k^{\frac{5}{3}}$. Resolutions $N = D/\Delta = 256$; 128, red and green respectively. c. Resolution $N = 64$. The large-scale spectra are independent on Δ. Fluctuations are sensitive.

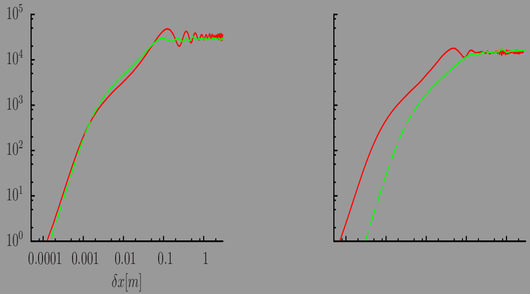
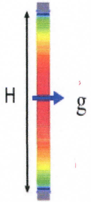
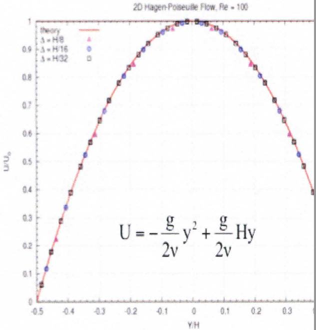


Figure: left: Second-order structure function $S_2 = \overline{|u(x+\delta x) - u(x)|^2} \propto |\delta x|^{\frac{2}{3}}$. Inertial+analytic+energy ranges are there. right: Third-order structure function $S_3 = \overline{|u(x+\delta x) - u(x)|^3} \propto |\delta x|$. Resolutions $N = 256$ and $N = 128$, respectively. S_3 is much more sensitive. At the integral scale $S_n \to 2\overline{v^n} = const$.

XXXX

Gravity Driven Hagen-Poiseuille Flow





$$U = -\frac{g}{2\nu} y^2 + \frac{g}{2\nu} H y$$

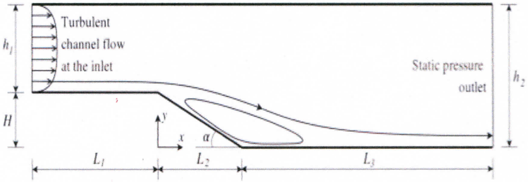
Gravity driven Hagen-Poiseuille Flow

Channel Height :	H
Resolution:	N
Mean Velocity:	$\overline{U}$
Max. Velocity:	$U_o = 1.5\overline{U}$
Viscosity:	ν
Reynolds Number :	100

$$\overline{U} = \frac{gH^2}{12\nu}; \quad U_o = \frac{gH^2}{8\nu}$$

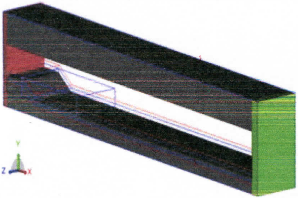
$$Re = \frac{\overline{U}H}{\nu}, \qquad \text{voxel size} : \Delta = \frac{H}{N}$$

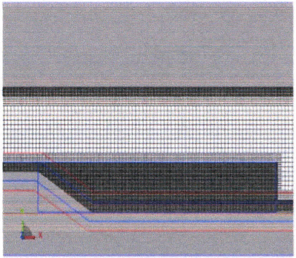
Flow Setup - Inclined Back Steps



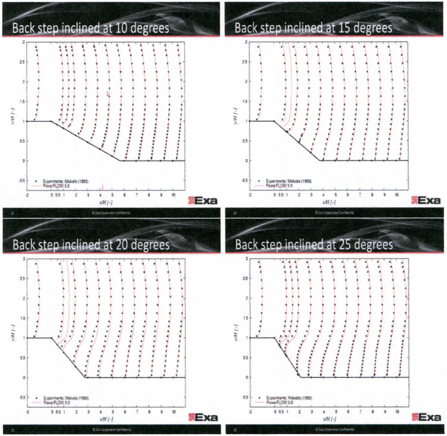
- Case 53 of the European Research Community on Flow, Turbulence and Combustion (ERCOFTAC) database
- Based on closed-loop WT experiments of Ruck & Makiola (1993)
- Reynolds number based on step height H is 64,000.
- Expansion ratio, $ER = h_2/h_1 = 1.48$
- Inclinations of 10, 15, 20, 25, 30 and 90 degrees were studied

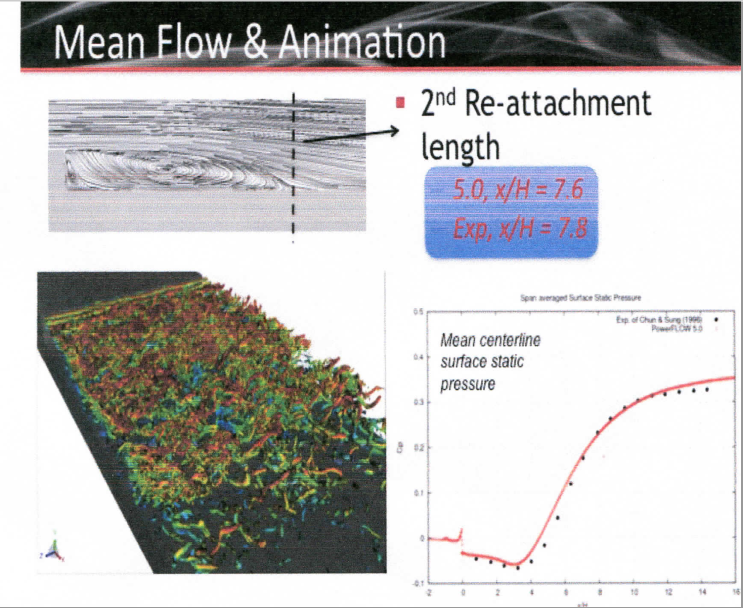
VR Setup - Inclined Back Steps

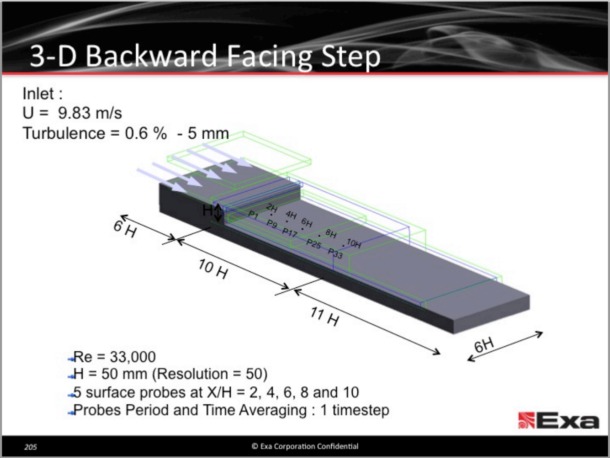




- 3 levels of VR
- Finest VR corresponds to H/Δ = 72


Back step inclined at 10 degrees
Back step inclined at 15 degrees
Back step inclined at 20 degrees
Back step inclined at 25 degrees
Exa
Exa
Exa
Exa


Mean Flow & Animation
2nd Re-attachment length
5.0, x/H = 7.6
Exp, x/H = 7.8
Span averaged Surface Static Pressure
Exp. of Chun & Sung (1996)
PowerFLOW 5.0
Mean centerline surface static pressure


3-D Backward Facing Step
Inlet :
U = 9.83 m/s
Turbulence = 0.6 % - 5 mm
H
6 H
10 H
11 H
6H
2H
4H
6H
8H
10H
P1
P9
P17
P25
P33
Re = 33,000
H = 50 mm (Resolution = 50)
5 surface probes at X/H = 2, 4, 6, 8 and 10
Probes Period and Time Averaging : 1 timestep
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205
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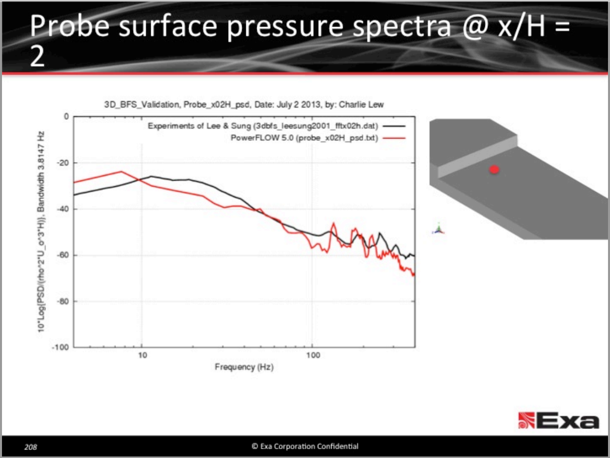
Probe surface pressure spectra @ x/H = 2
3D_BFS_Validation, Probe_x02H_psd, Date: July 2 2013, by: Charlie Lew
Experiments of Lee & Sung (3dbfs_leesung2001_fftx02h.dat)
PowerFLOW 5.0 (probe_x02H_psd.txt)
10*Log(PSD/(rho^2*U_o^3*H)), Bandwidth 3.8147 Hz
0
-20
-40
-60
-80
-100
10
100
Frequency (Hz)
Exa
208
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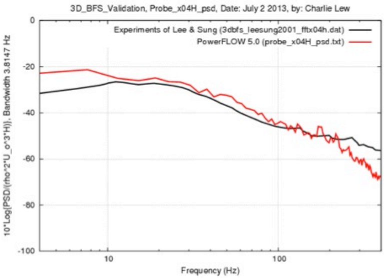
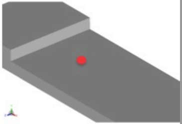
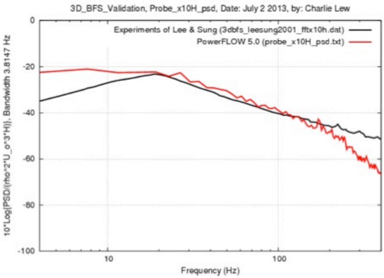
Probe surface pressure spectra @ x/H = 4

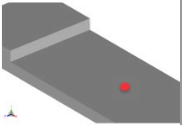




Exa

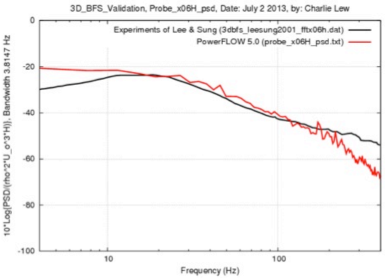
Probe surface pressure spectra @ x/H = 10

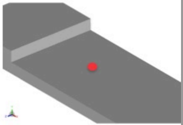




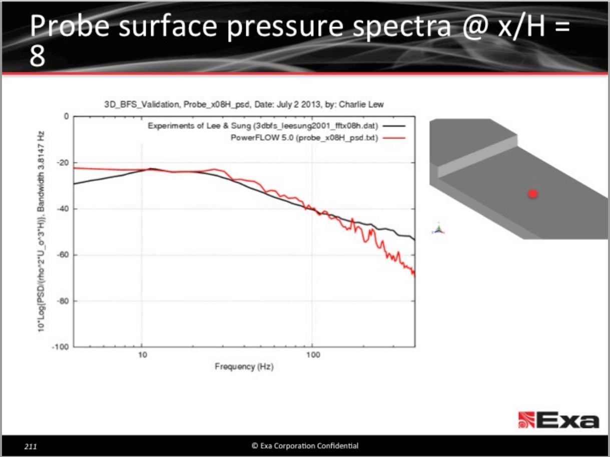
Exa

Probe surface pressure spectra @ x/H = 6





Exa


Probe surface pressure spectra @ x/H = 8
3D_BFS_Validation, Probe_x08H_psd, Date: July 2 2013, by: Charlie Lew
Experiments of Lee & Sung (3dbfs_leesung2001_fftx08h.dat)
PowerFLOW 5.0 (probe_x08H_psd.txt)
10*Log(PSD/(rho^2*U_o^3*H)), Bandwidth 3.8147 Hz
0
-20
-40
-60
-80
-100
10
100
Frequency (Hz)
Exa
211
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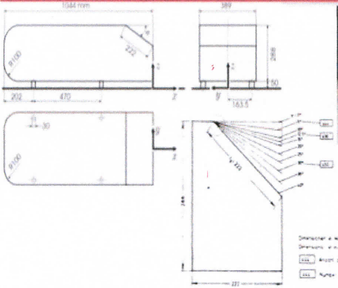
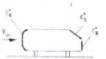
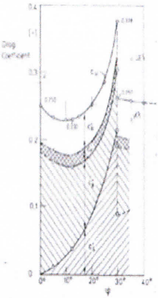
Ahmed Body



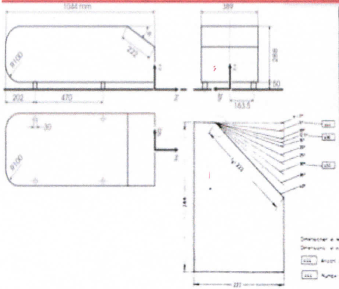


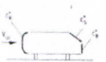


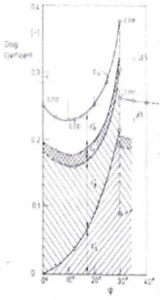
From Ahmed, Ramm & Faltin (1984)

- Original drag measurements by Ahmed et al. (1984)
- Lienhart et al. (2004) performed LDA measurements at 25 and 35 degrees, did not measure drag

Ahmed Body

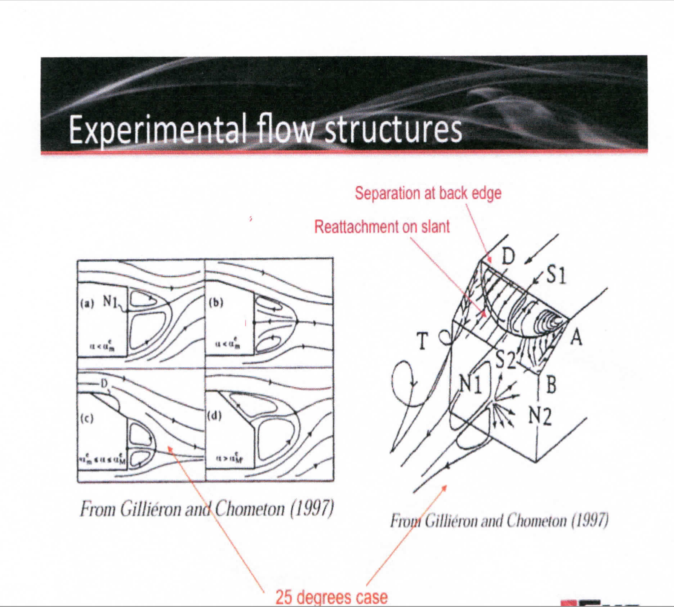




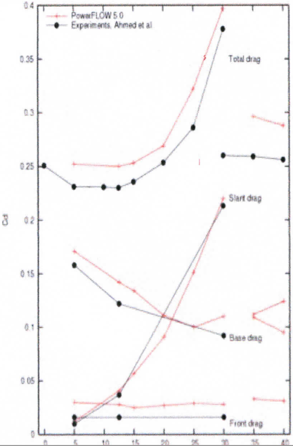


From Ahmed, Ramm & Faltin (1984)

- Original drag measurements by Ahmed et al. (1984)
- Lienhart et al. (2004) performed LDA measurements at 25 and 35 degrees, did not measure drag


Experimental flow structures
Separation at back edge
Reattachment on slant
(a) N1
(b)
(c)
(d)
D
From Gilliéron and Chometon (1997)
D
S1
T
A
S2
N1
B
N2
From Gilliéron and Chometon (1997)
25 degrees case

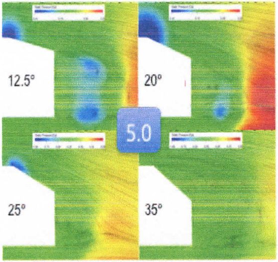
Drag curve

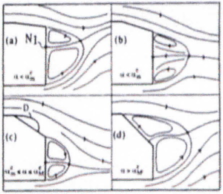


PowerFLOW 5.0 shows an excellent agreement with the overall drag and the component drag measurements of Ahmed *et al.* slant angles.

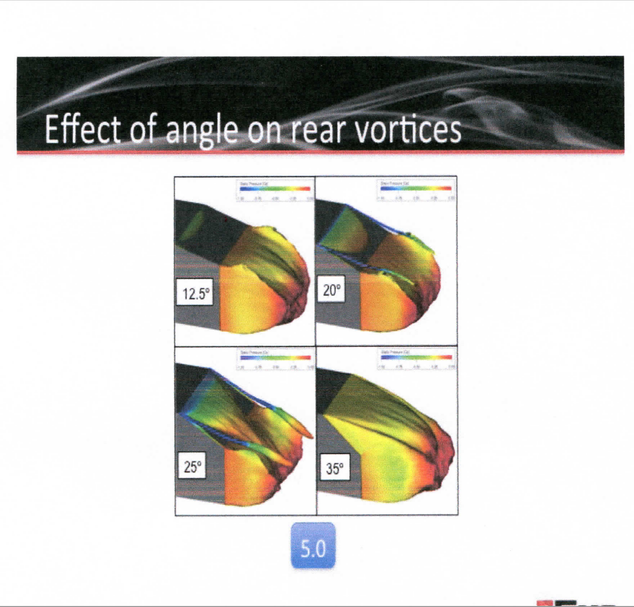
The drag at 25° is slightly high, however flow structures at this angle are in excellent agreement with those measured by Leinhart *et al.*

Effect of angle on rear separation

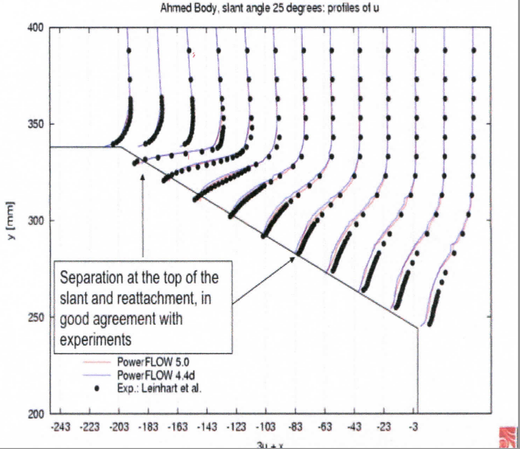


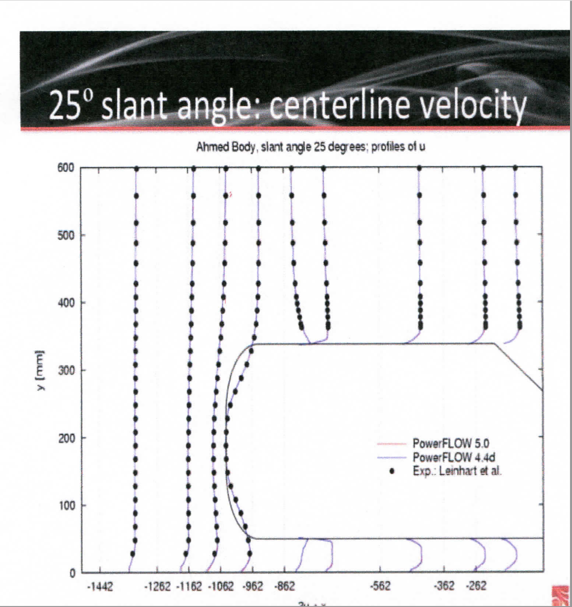


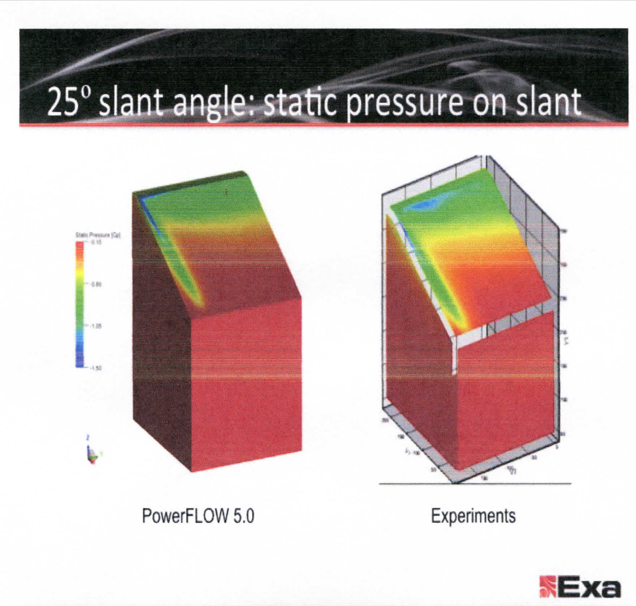
From Gilliéron and Chometon (1997)


Effect of angle on rear vortices
12.5°
20°
25°
35°
5.0

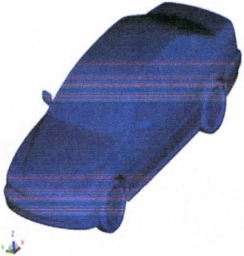
25° slant angle: centerline velocity on slant



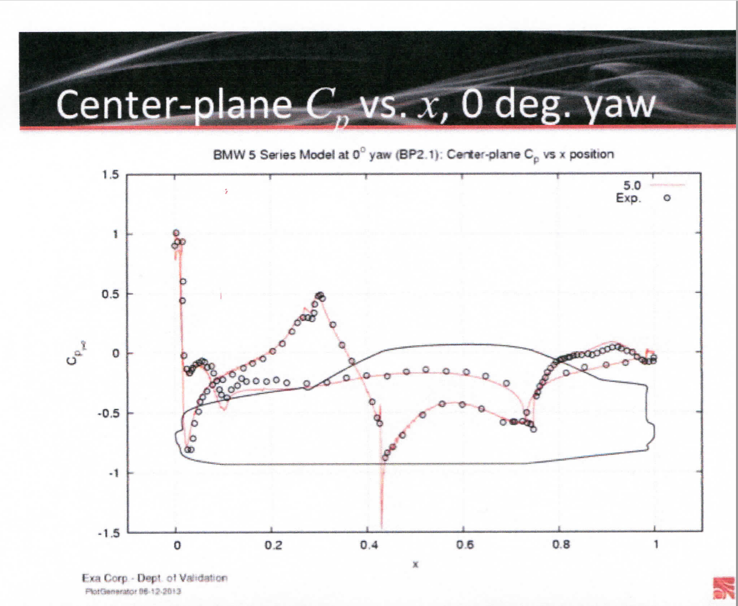

25° slant angle: centerline velocity
Ahmed Body, slant angle 25 degrees; profiles of u
600
500
400
300
200
100
0
y [mm]
-1442
-1262
-1162
-1062
-962
-862
-562
-362
-262
PowerFLOW 5.0
PowerFLOW 4.4d
Exp.: Leinhart et al.

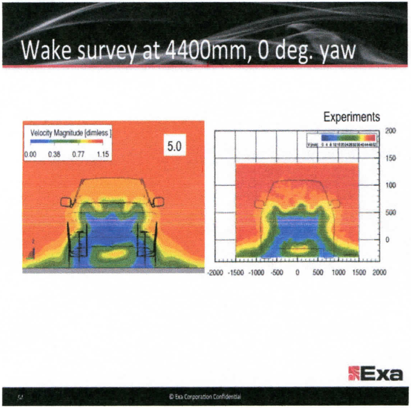

25° slant angle: static pressure on slant
PowerFLOW 5.0
Experiments
Exa

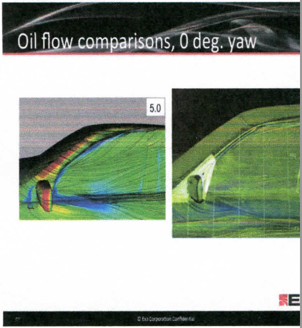
- BMW 530i model
- Part of the EADE study
- 45 m/s (162 km/hr)
- 0 and 10 degree yaw cases

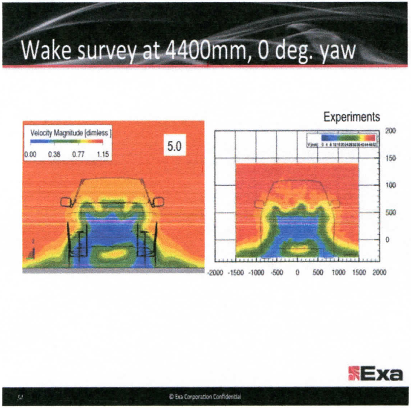


- Best Practices 2.1 setup
  - *21M FEVos for 0 degree yaw case*
  - *Modified VR6,7,8 for 10 degree yaw case to enclose larger A-pillar and trunk vortices (22M FEVos)*


Center-plane C_p vs. x, 0 deg. yaw
BMW 5 Series Model at 0° yaw (BP2.1): Center-plane C_p vs x position
5.0
Exp.
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
x
Exa Corp - Dept. of Validation


Wake survey at 4400mm, 0 deg. yaw
Velocity Magnitude [dimless]
0.00 0.38 0.77 1.15
5.0
Experiments
200
150
100
500
0
-2000 -1500 -1000 -500 0 500 1000 1500 2000
Exa
© Exa Corporation Confidential


Oil flow comparisons, 0 deg. yaw
5.0
© Exa Corporation Confidential


Wake survey at 4400mm, 0 deg. yaw
Velocity Magnitude [dimless]
0.00 0.38 0.77 1.15
5.0
Experiments
200
150
100
500
0
-2000 -1500 -1000 -500 0 500 1000 1500 2000
Exa
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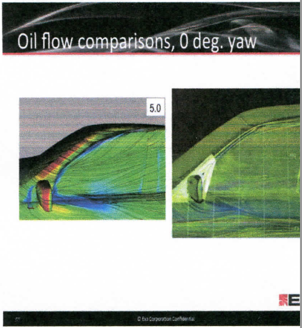
Oil flow comparisons, 0 deg. yaw
5.0
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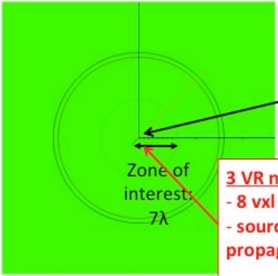
Case setup: Monopole Source & Resolution



3 VR near source:
- **8 vxl each**
- **source resolution = propagation resolution**

3D monopole source:
-Centered at (0,0,0)
-Frequency: f = 1000 Hz
-Wavelength: $\lambda = c_0/f$
with c_0 the speed of sound
-Defined as solid sphere of radius **$r=\lambda/8$**

boundary condition
-Inlet: P&u
where
$P = P_0 + A*\cos(2\pi f*t)$
$u = A/\cos\theta *\cos(2\pi f*t - \theta)/\rho c$
with A = 100 Pa
$\cot\theta = 2\pi r/\lambda$
3 Symmetry planes

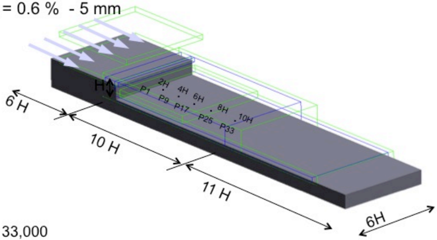
runtime: 0.12 seconds

Initial condition (IC): no flow
-U_{IC} = 0 m/s
-$P_{IC} = P_0$
-$\rho_{IC} = \rho_0$

Resolution:
-Resolution = number of points per λ

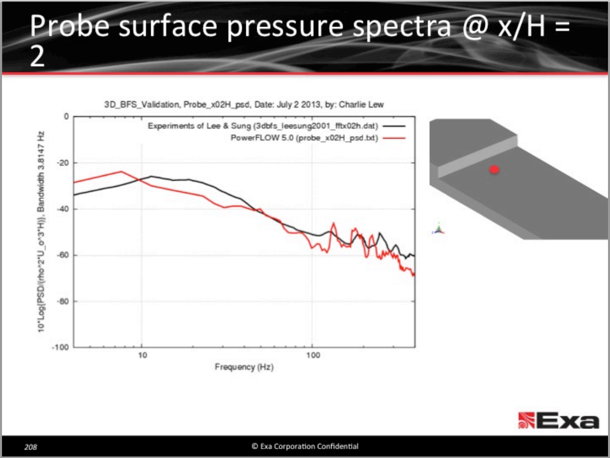
3-D Backward Facing Step

Inlet :
U = 9.83 m/s
Turbulence = 0.6 % - 5 mm



- Re = 33,000
- H = 50 mm (Resolution = 50)
- 5 surface probes at X/H = 2, 4, 6, 8 and 10
- Probes Period and Time Averaging : 1 timestep

Exa


Probe surface pressure spectra @ x/H = 2
3D_BFS_Validation, Probe_x02H_psd, Date: July 2 2013, by: Charlie Lew
Experiments of Lee & Sung (3dbfs_leesung2001_fftx02h.dat)
PowerFLOW 5.0 (probe_x02H_psd.txt)
10*Log(PSD/(rho^2*U_o^3*H)), Bandwidth 3.8147 Hz
Frequency (Hz)
0
-20
-40
-60
-80
-100
10
100
Exa
208
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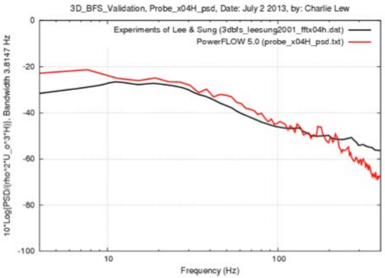
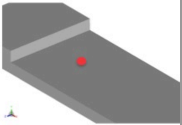
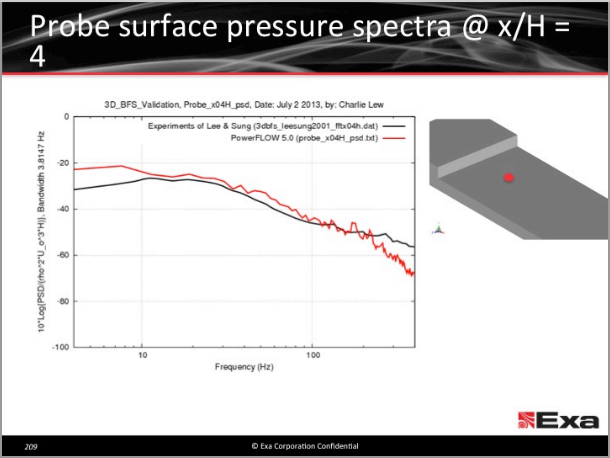
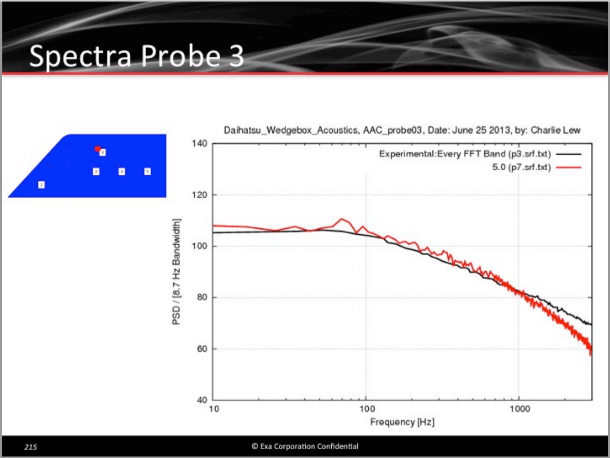
Probe surface pressure spectra @ x/H = 4





Universality of turbulence and engineering simulations.




Spectra Probe 3
Daihatsu_Wedgebox_Acoustics, AAC_probe03, Date: June 25 2013, by: Charlie Lew
Experimental:Every FFT Band (p3.srf.txt)
5.0 (p7.srf.txt)
PSD / [8.7 Hz Bandwidth]
Frequency [Hz]
140
120
100
80
60
40
10
100
1000
215
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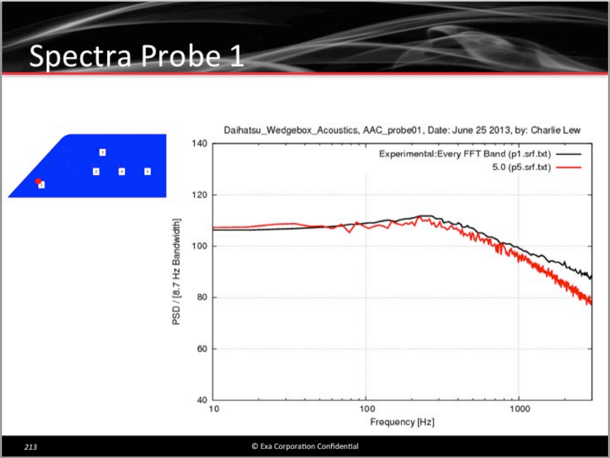
Spectra Probe 1
Daihatsu_Wedgebox_Acoustics, AAC_probe01, Date: June 25 2013, by: Charlie Lew
Experimental:Every FFT Band (p1.srf.txt)
5.0 (p5.srf.txt)
PSD / [8.7 Hz Bandwidth]
140
120
100
80
60
40
10
100
1000
Frequency [Hz]
213
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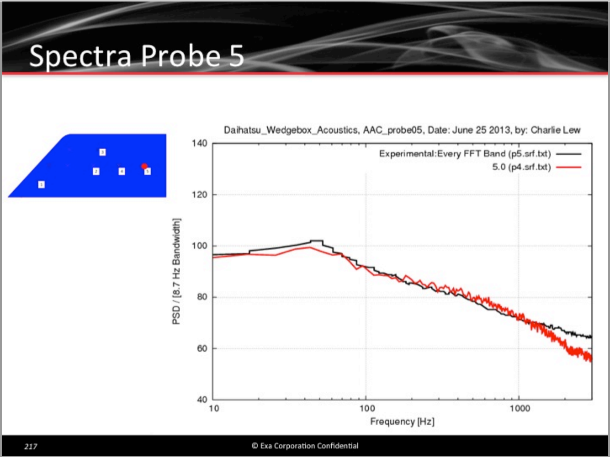
Spectra Probe 5
Daihatsu_Wedgebox_Acoustics, AAC_probe05, Date: June 25 2013, by: Charlie Lew
Experimental:Every FFT Band (p5.srf.txt)
5.0 (p4.srf.txt)
PSD / [8.7 Hz Bandwidth]
Frequency [Hz]
140
120
100
80
60
40
10
100
1000
217
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