PRINCETON UNIVERSITY

Experimental investigations of the scaling of turbulence in pipes and channels

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High Reynolds Number Boundary Layer Turbulence: Integrating Descriptions of Statistical Structure, Scaling and Dynamical Evolution
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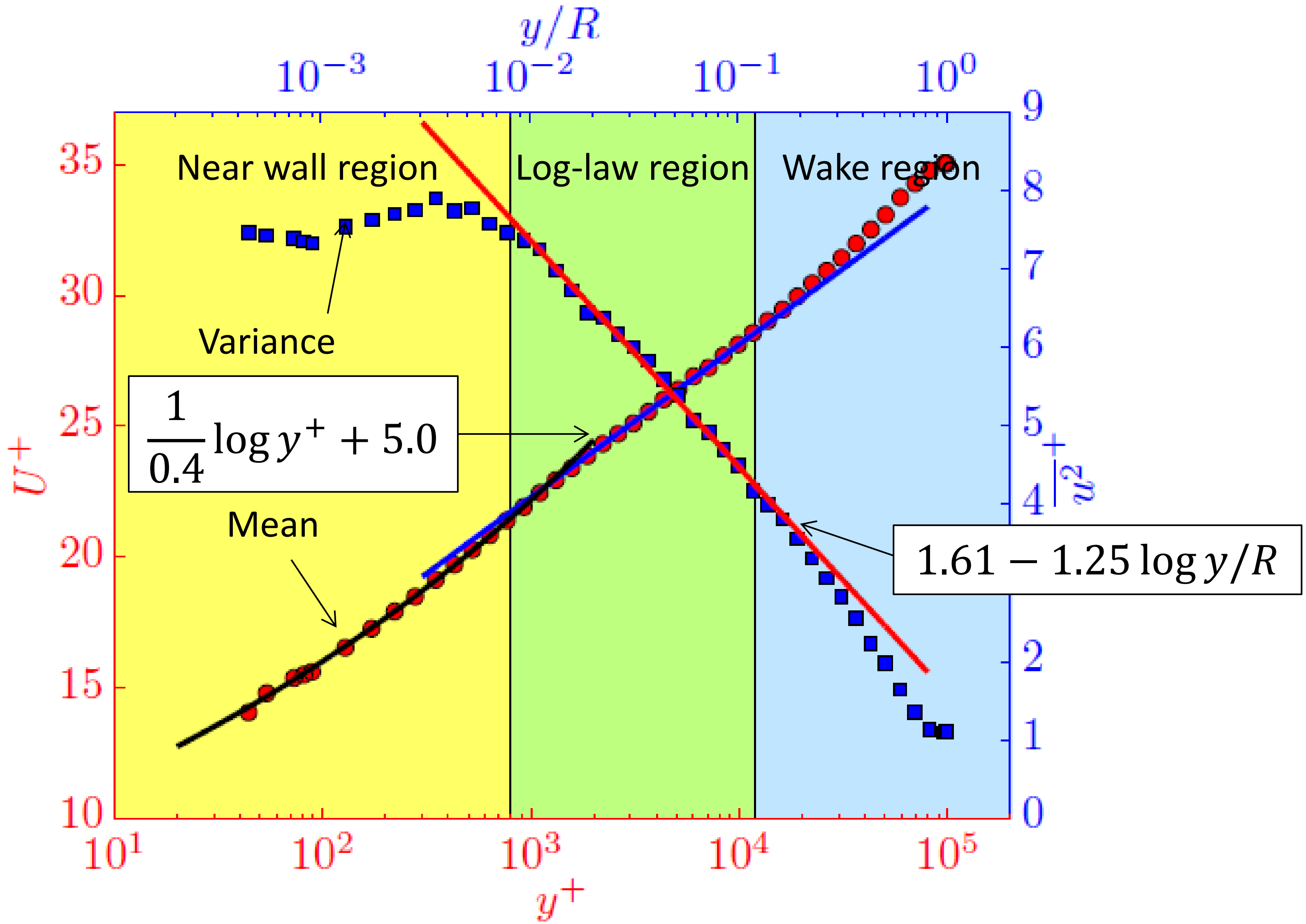
Duality between fluctuations and mean velocities



Hultmark, Vallikivi, Bailey and Smits (PRL) 2012

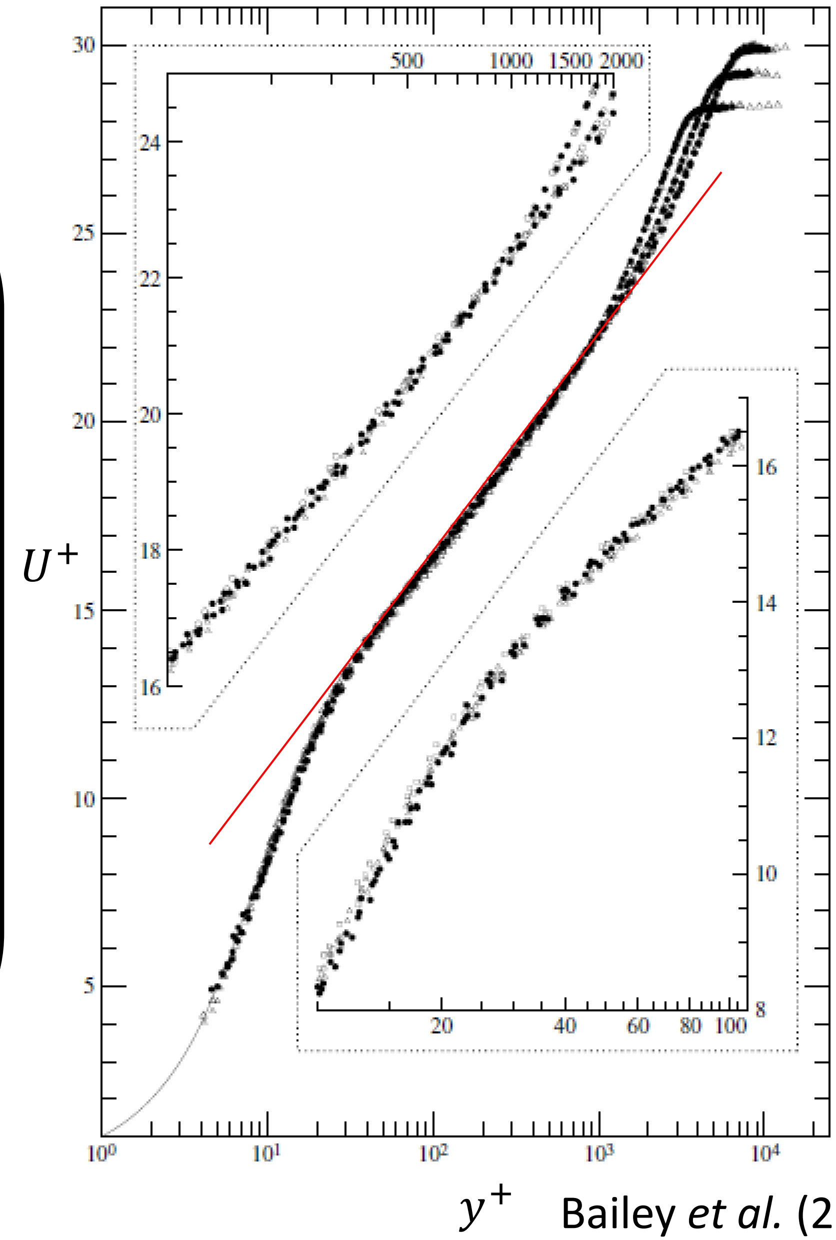
Mean flow overlap argument

Inner: $$U^+ = f(y^+)$$

Outer: $$\frac{U_\infty - U}{u_\tau} = g\left(\frac{y}{R}\right)$$

$$\longrightarrow \quad U^+ = \frac{1}{\kappa}\ln(y^+) + B$$

(log-law in the overlap region)



Bailey *et al.* (2013)

Two problems with the classical scaling

- Only strictly valid at infinite Reynolds numbers.
- For mean velocities only, cannot explain the duality observed between the mean velocity and the fluctuations.

Two problems with the classical scaling

- Only strictly valid at infinite Reynolds numbers.

- Wosnik *et al.* (2000) extended it to finite Reynolds numbers by allowing the functions to vary with Reynolds number.

$$U^+ = f_i(y^+, R^+)$$

$$\frac{U_\infty - U}{u_\tau} = f_o(\bar{y}, R^+)$$

by introducing an intermediate variable $\tilde{y} = y^+ {R^+}^{-n}$ and by differentiating with respect to R^+ while keeping $\tilde{y}$ constant (following George & Castillo 1997), they found that:

$$\bar{y}\frac{\partial f_o}{\partial \bar{y}}\bigg|_{R^+} = \frac{1}{\kappa(R^+)} + \left[\frac{\partial f_i(y^+, R^+)}{\partial \log(R^+)}\bigg|_{y^+} - \frac{\partial f_o(\bar{y}, R^+)}{\partial \log(R^+)}\bigg|_{\bar{y}}\right] = \frac{1}{\kappa(R^+)} + S_m$$

- If there exists a region in space where $S_m = 0$, the mean velocities will exhibit a the logarithmic law. Also note that $S_m = constant$ would result in a logarithm.
- $\frac{1}{\kappa} = \frac{dU_{cl}^+}{d\log R^+}$
- $U^+ = \frac{1}{\kappa}\ln(y^+ + a^+) + B$

Two problems with the classical scaling

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- Wosnik et al. (2000) dealt with finite Reynolds numbers by allowing the functions vary with Reynolds number.

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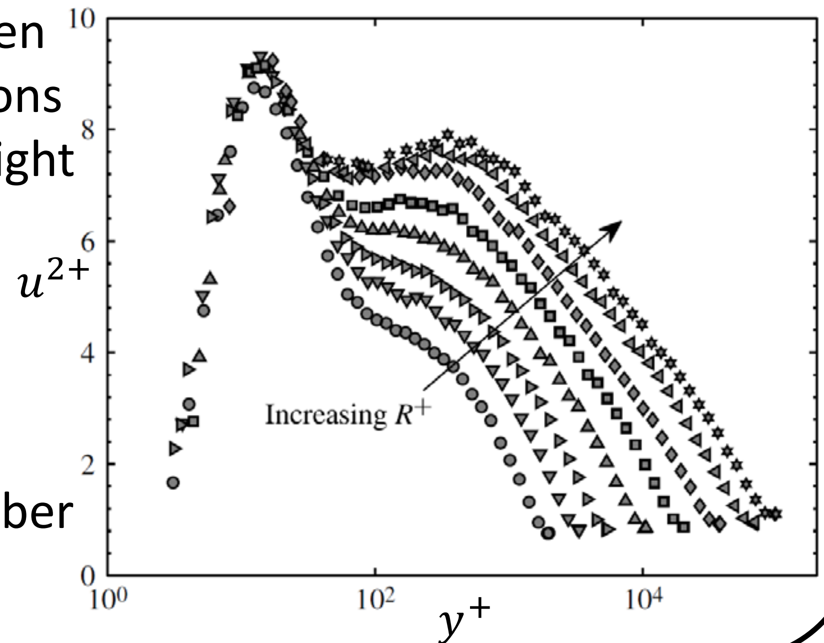
- If $S_m = 0$ then we recover the logarithmic law.
- $\frac{1}{\kappa} = \frac{dU_{cl}^+}{d\log R^+}$
- $U^+ = \frac{1}{\kappa}\ln(\mathrm{y}^+ + a^+) + \mathrm{B}$

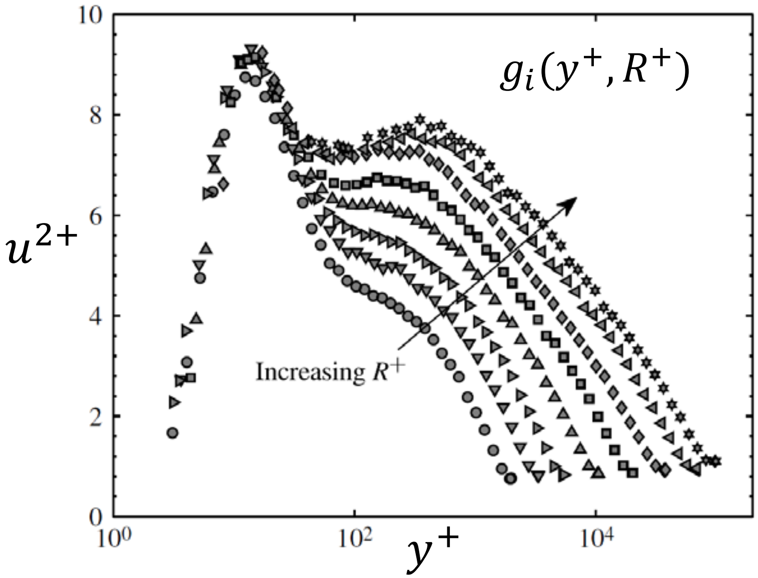
Two problems with the classical scaling

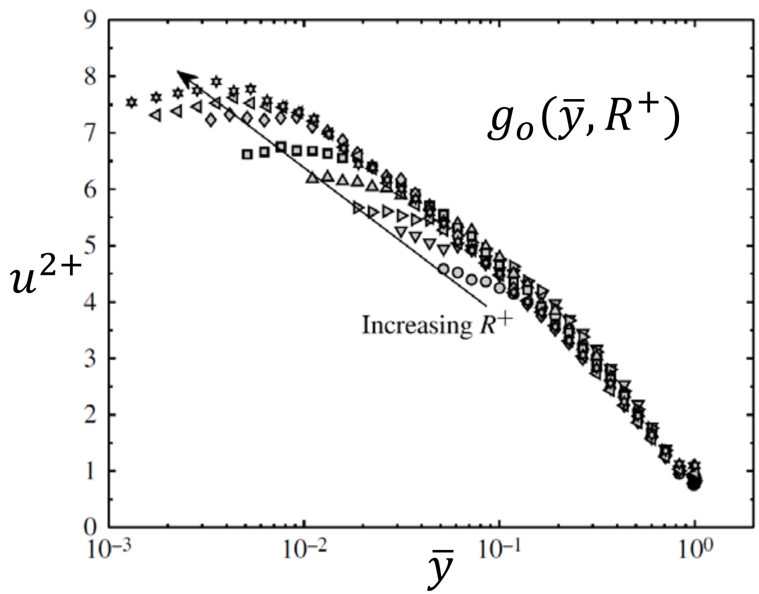
- Only strictly valid at infinite Reynolds numbers.
- Does only work for the mean velocity, cannot explain the duality observed between the mean velocity and the fluctuations.

- The as good as perfect match between the logarithmic layer in the fluctuations and the mean suggests that there might be a matching theory.
- **Problem:** No obvious offset in the variances (centerline velocity for the mean velocities, $(U_\infty - U)/u_\tau$)
- There will always be a Reynolds number trend in the fluctuations.



Reynolds number dependence in the fluctuations





Can extend the approach by Wosnik et al. (2000)

$$u^{2+} = g_i(y^+, R^+)$$

$$u^{2+} = g_o(\bar{y}, R^+)$$

introducing an intermediate variable $\tilde{y} = y^+ {R^+}^{-n}$ and by differentiating with respect to R^+ while keeping $\tilde{y}$ constant, we find.

$$\bar{y}\frac{\partial g_o}{\partial \bar{y}}\bigg|_{R^+} = \left[\frac{\partial g_i(y^+, R^+)}{\partial \log(R^+)}\bigg|_{y^+} - \frac{\partial g_o(\bar{y}, R^+)}{\partial \log(R^+)}\bigg|_{\bar{y}}\right] = -S_f$$

If S_f is constant anywhere in space we can expect the profile to be logarithmic in the same region. And the slope of the logarithm will be $-S_f$.

$$u^{2+} = B_0 - S_f \log(\bar{y} + b^+)$$

Hultmark (JFM) 2012

Sensitivity functions for the mean and the fluctuations

Evaluate the sensitivity functions by interpolating the data at $Re_\tau = 98{,}000$ and $Re_\tau = 37{,}000$ to match y^+ and $\bar{y}$ of $Re_\tau = 68{,}000$ and evaluate the gradients in Reynolds number.

$$S_f = -\left[\left.\frac{\partial g_i(y^+, R^+)}{\partial \log(R^+)}\right|_{y^+} - \left.\frac{\partial g_o(\bar{y}, R^+)}{\partial \log(R^+)}\right|_{\bar{y}} \right]$$

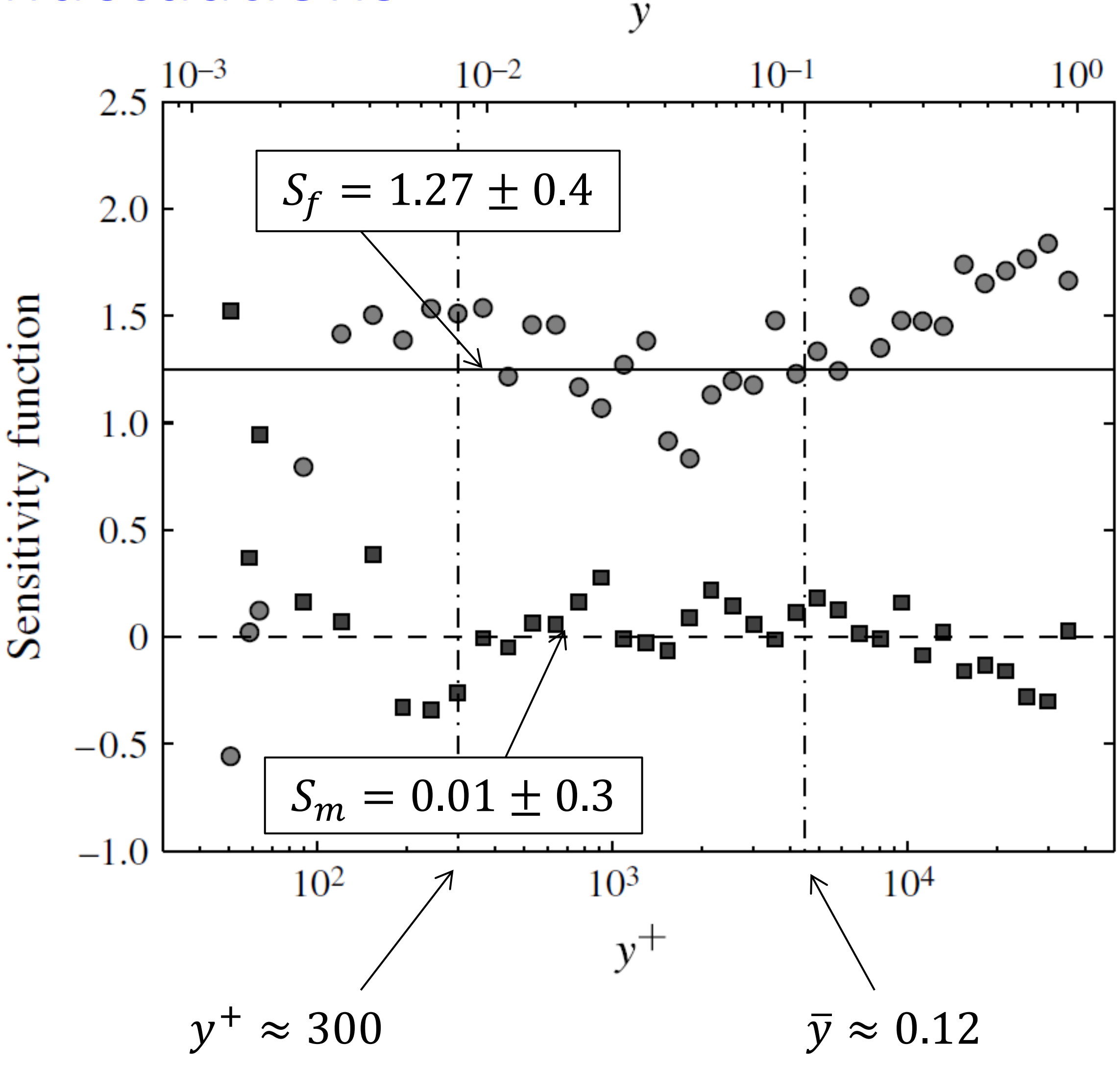
$$S_m = \left[\left.\frac{\partial f_i(y^+, R^+)}{\partial \log(R^+)}\right|_{y^+} - \left.\frac{\partial f_o(\bar{y}, R^+)}{\partial \log(R^+)}\right|_{\bar{y}} \right]$$

Can expect:

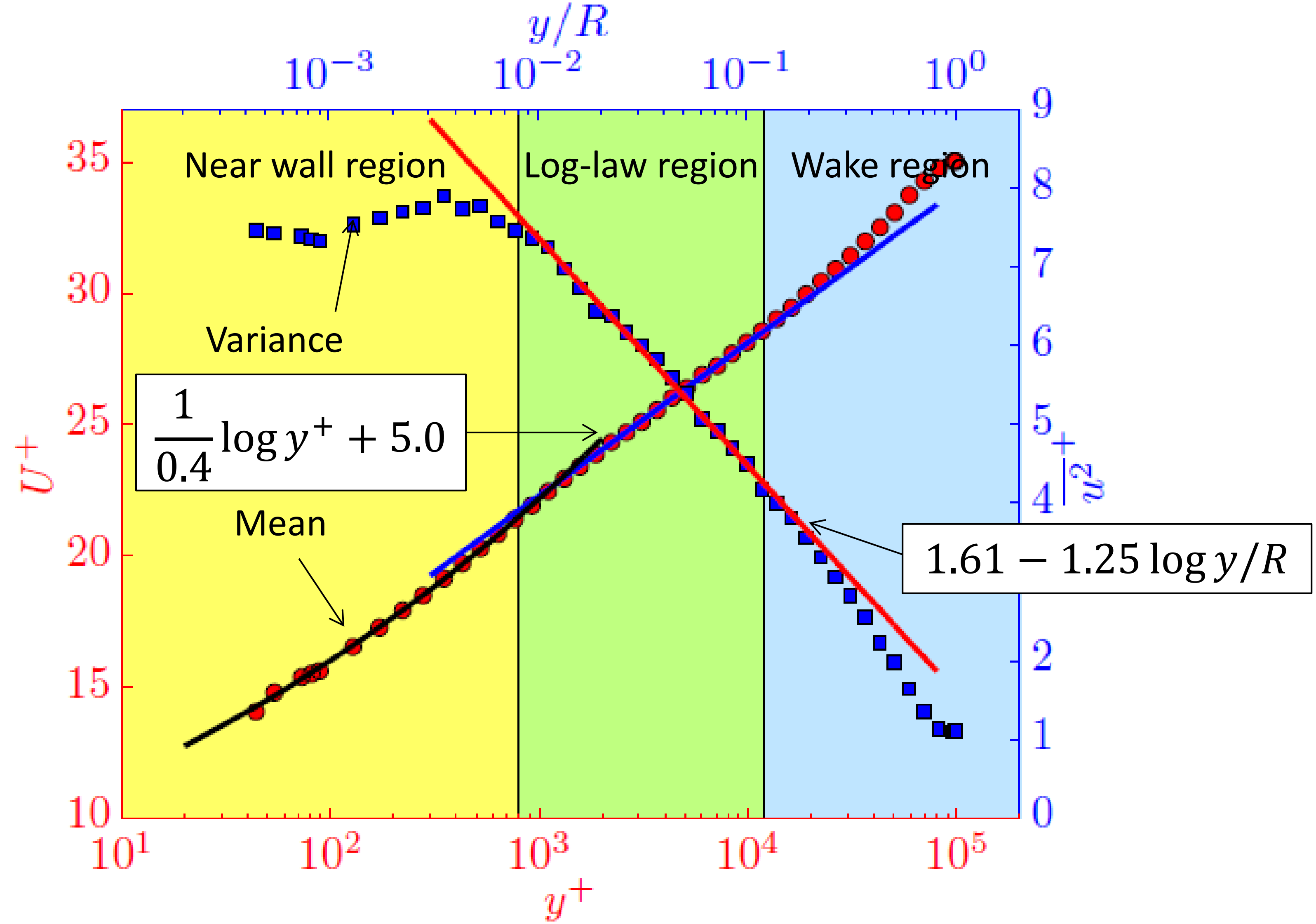
$$u^{2+} = B_1 - 1.27\log(\bar{y} + \bar{b})$$

and

$$U^+ = \frac{1}{\kappa}\log(y^+ + a^+) + B$$



Duality between fluctuations and mean velocities

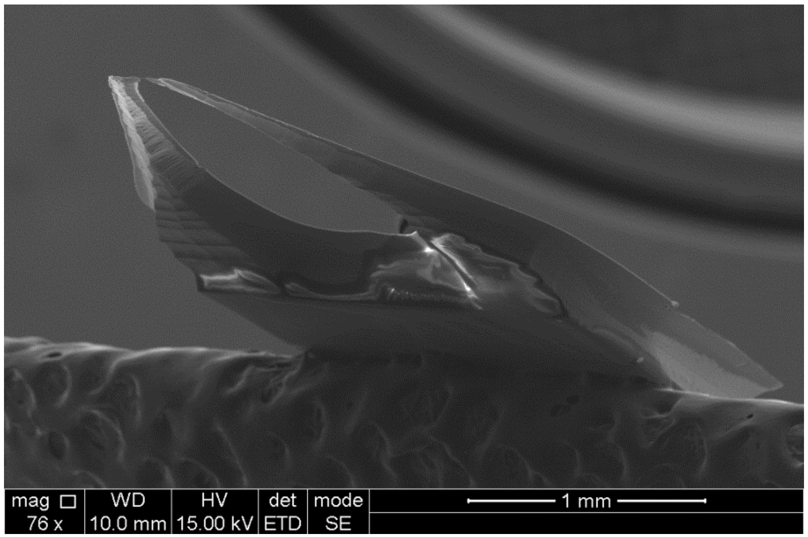


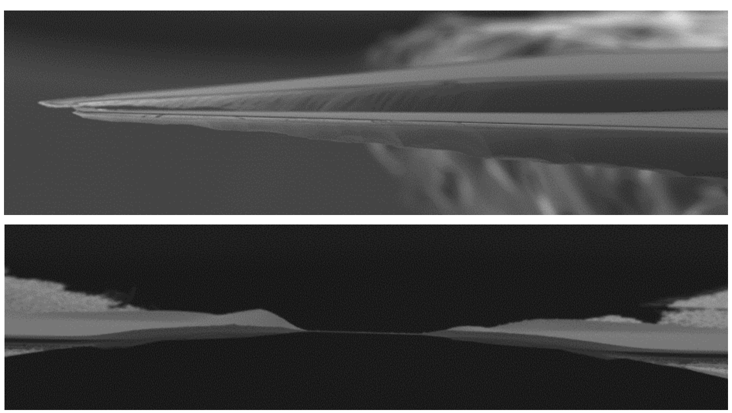
Hultmark, Vallikivi, Bailey and Smits (PRL) 2012

New sensors made it possible (together with a very special facility)

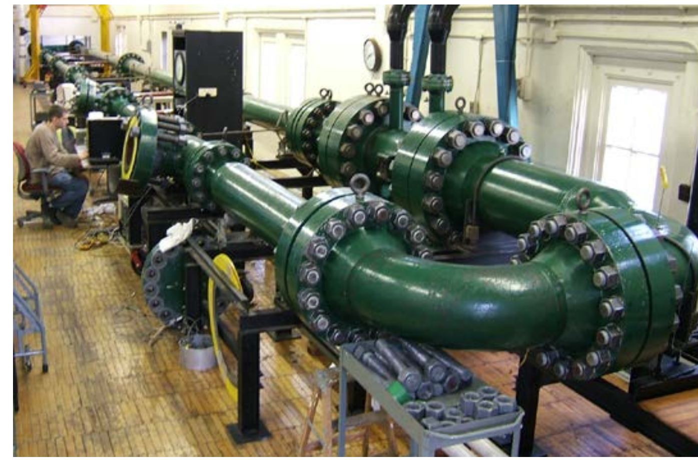
Nano-Scale Thermal Anemometer Probe (NSTAP)

Bailey *et al.* (2010), Vallikivi *et al.* (2011) , Vallikivi and Smits (under review)





Superpipe



Zagarola and Smits (1997)

- The NSTAP is more than one order of magnitude smaller than regular hot wires (improved spatial resolution)
- Improved temporal resolution ~150kHz
- **Well resolved turbulence measurements up to $Re_\tau = 100,000$.**

Similar scaling for passive scalars?
What is needed for a detailed investigation?

Scaling of passive scalar in wall bounded turbulence

$$\frac{\partial \theta}{\partial t} + u_j \frac{\partial \theta}{\partial x_j} = \alpha \nabla^2 \theta$$

$$\theta = \Theta + \theta'$$

$$u_j = U_j + u'_j$$

U, Θ: mean components

u_j', θ': fluctuations

α: scalar diffusivity

$$\frac{\partial \Theta}{\partial t} + U \frac{\partial \Theta}{\partial x} = \alpha \frac{\partial^2 \Theta}{\partial y^2} - \frac{\partial \overline{v'\theta'}}{\partial y}$$

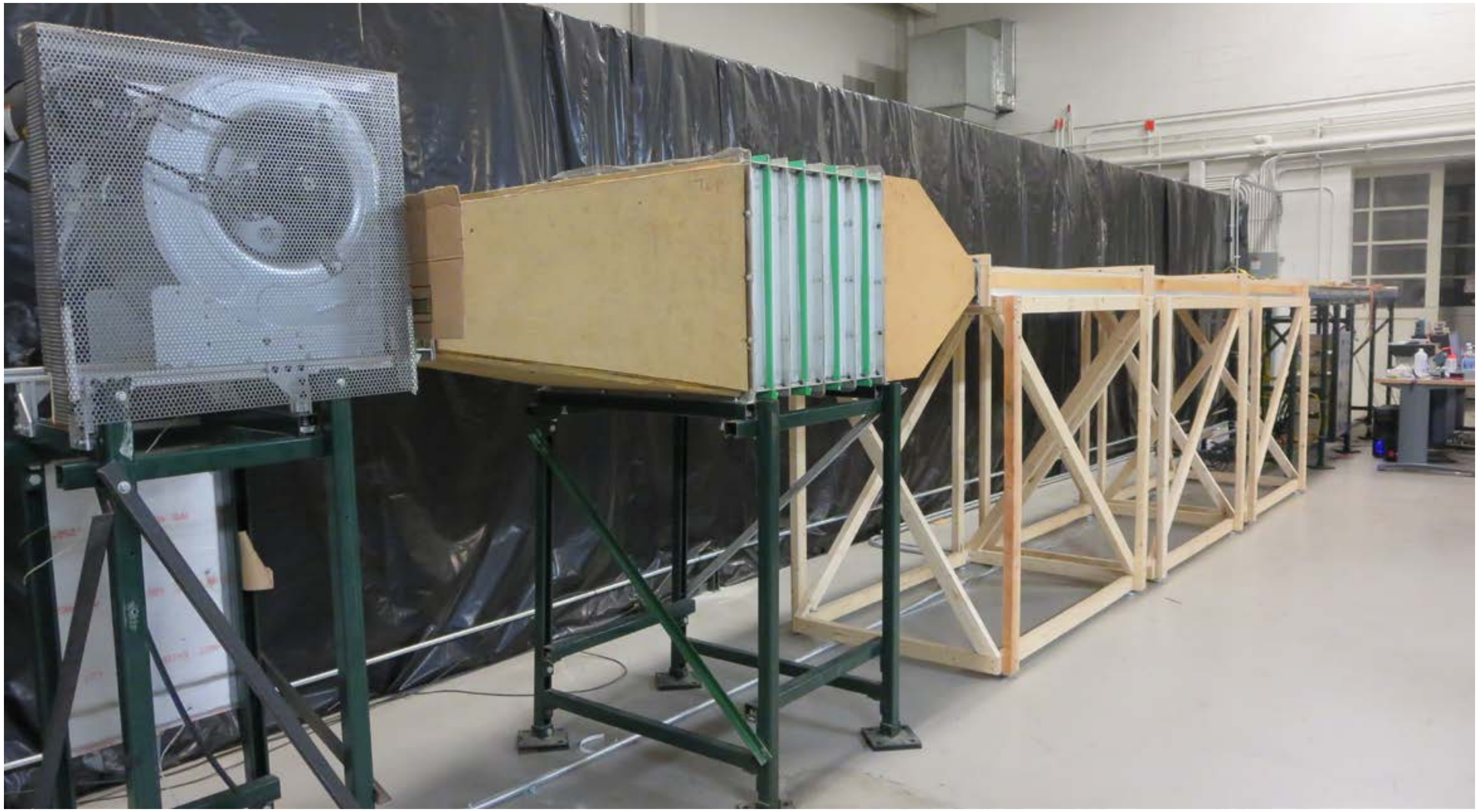
← Scalar flux

- Scalar flux: contribution of turbulence in the transport of the scalar
- Mean quantities alone are **not enough** to understand scalar transport in turbulent flows
- Knowledge about the **scalar flux** is needed

Two new facilities for temperature investigations

Channel flow

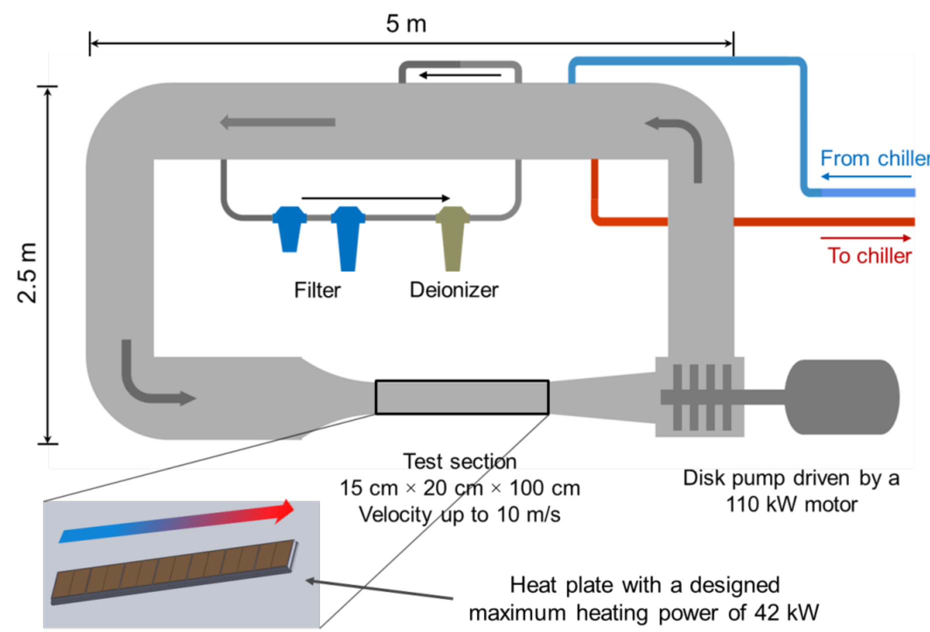
- Height $h = 2\delta = 6.35cm$
- Aspect ratio 12
- Unheated section: 5 meters ~80h
- Heated section: 4.75 meters ~75h



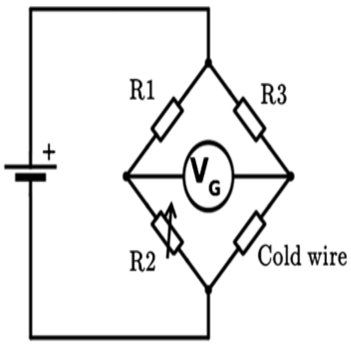
Two new facilities for temperature investigations

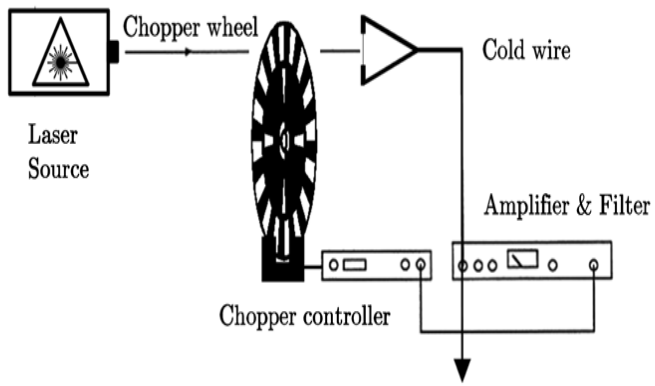
Water channel

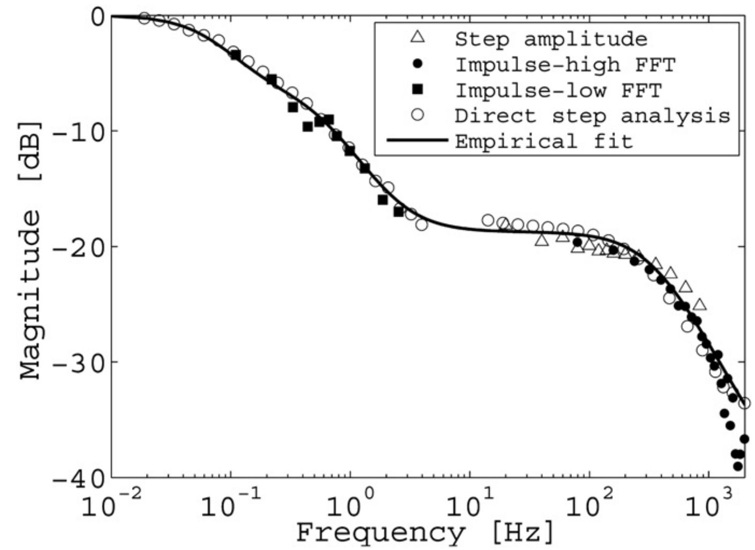
- 0.25 x 0.15 x 1 m test section
- Up to 13 m/s
- 30 kW of cooling power installed
- 42 kW of heating built into the wall of the test section for a developing thermal and velocity boundary layer
- De-ionizer



Evaluating of cold-wires



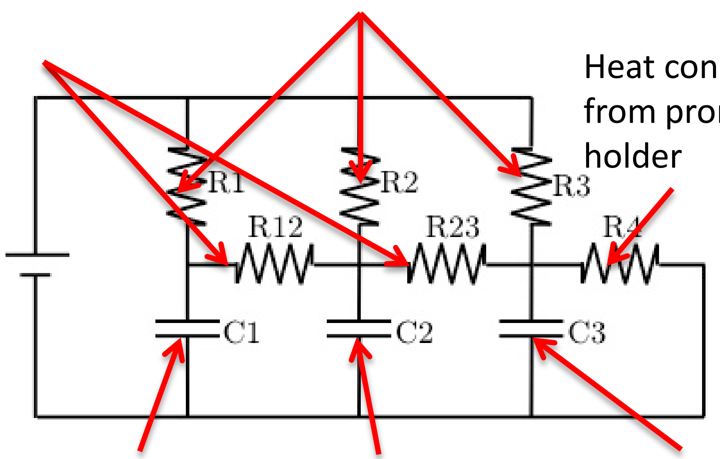




Conduction between elements

Heat transfer

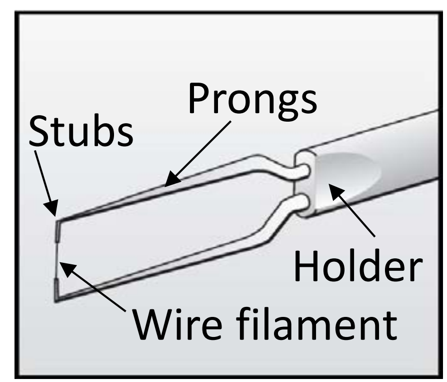
Heat conducted from prongs to holder



Heat accumulated in wire

Heat accumulated in stubs

Heat accumulated in prongs

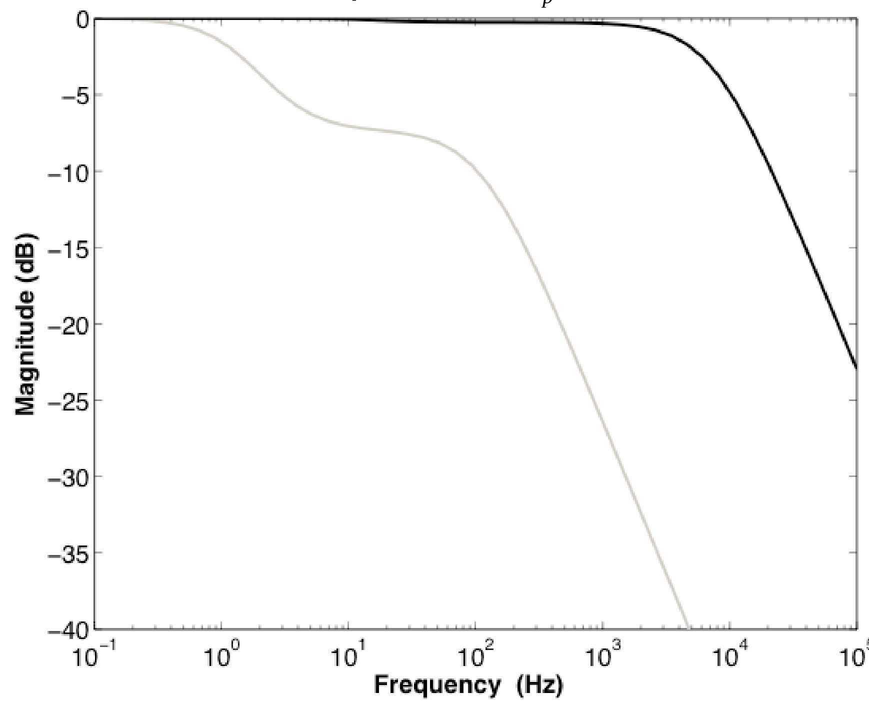


Arwatz *et al.* (MST) 2013

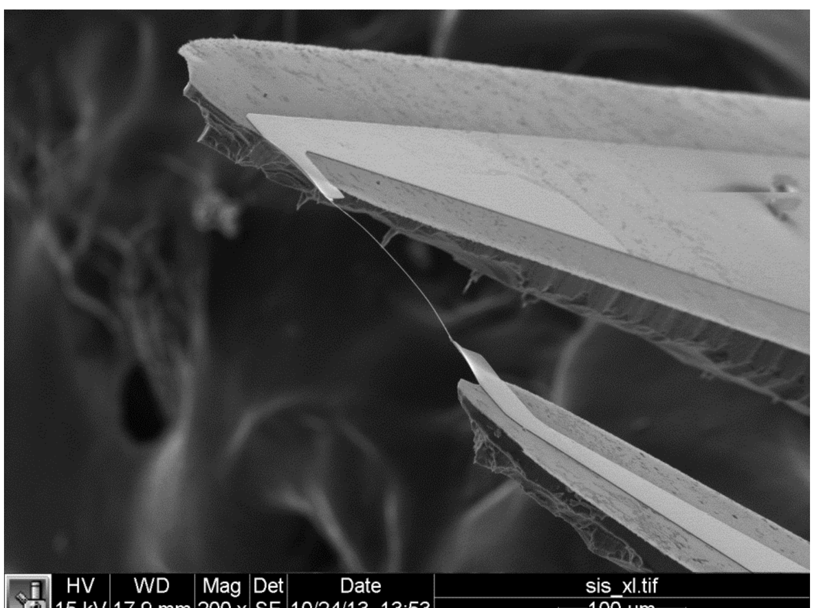
Design of true fast response temperature sensor

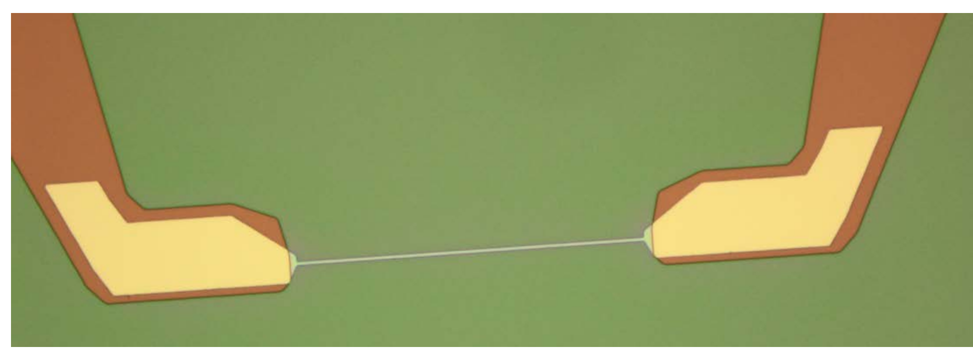
- Long and thin wire filament
- High conductivity prongs
- Thicker and shorter prongs

Wire filament	Gold prongs
$l = 300\mu m$	$l_p = 1mm$
$d = 200\eta m$	$d_p = 1mm$

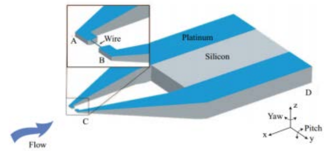


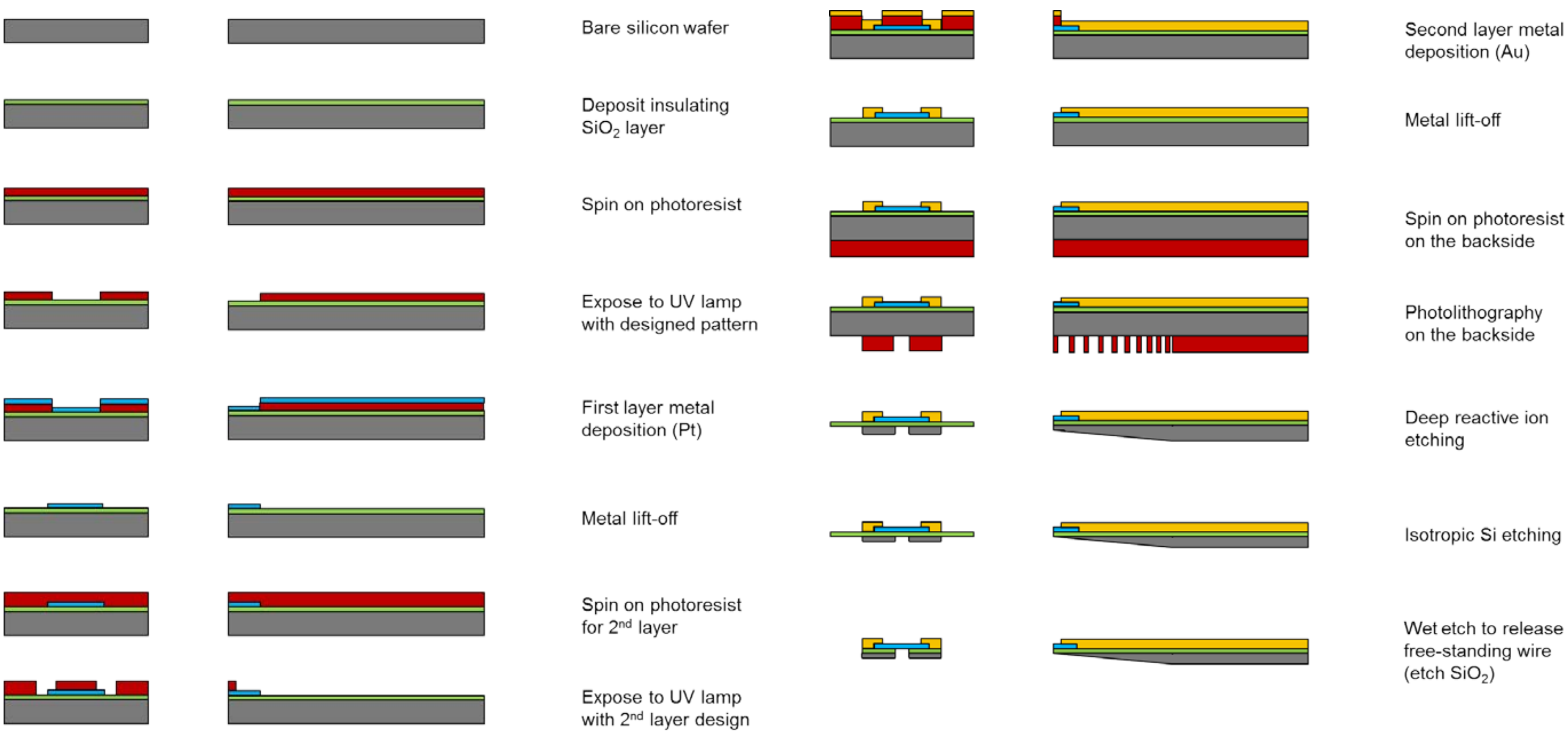
- Two metal construction
  platinum wire and gold stubs
- 200 x 0.1 x 1 μm



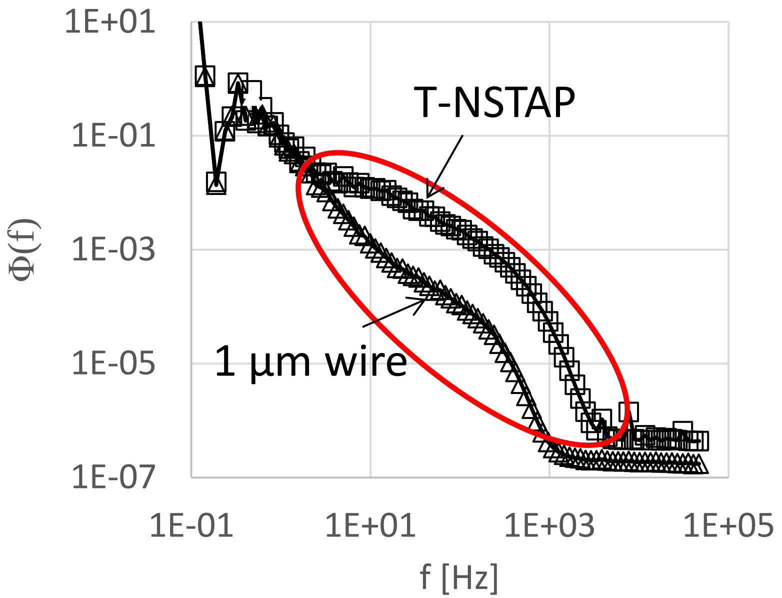


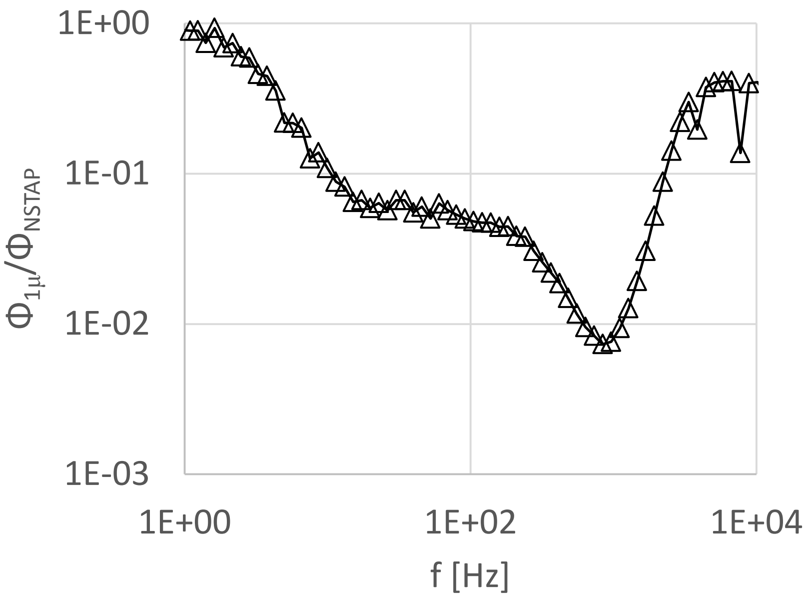
Fabrication process



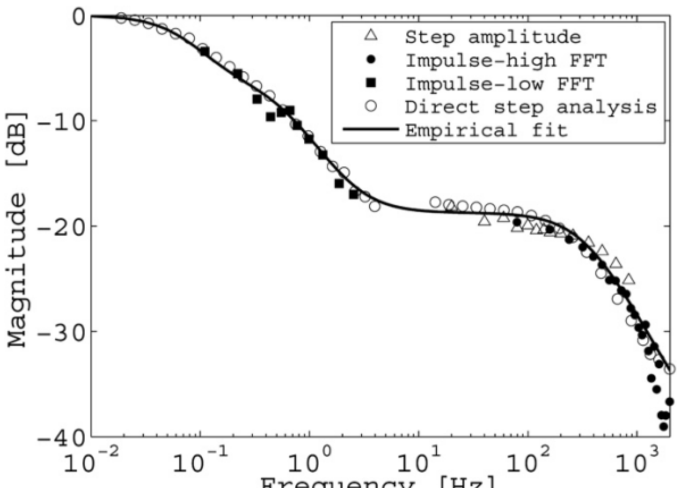


Measurement techniques – avoiding attenuation and resolving dissipative scales





- Temperature fluctuations measured with
  - T-NSTAP (100 nm thickness)
  - 1 µm wire (l/d=200)
- T-NSTAP has improved temporal and spatial resolution



Conclusions

- A new theory for the streamwise turbulent fluctuations in fully developed pipe flow was introduced.
- The slope of the logarithm of the variances relate to the derivative in Reynolds number.
- To investigate similar behavior for scalar fields two new facilities have been commissioned.
- A new fast response cold-wire is developed and tested