Dynamical systems approaches to
internal and external boundary layers

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Turbulence needs persistent
3-d structures as a scaffold
that supports the spatio-temporal
turbulent dynamics

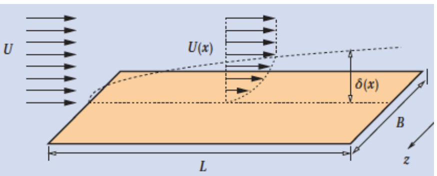
„Exact coherent structures"

Content

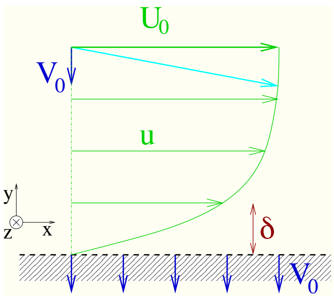
- Localized and extended edge states in the suction boundary layer
- Symmetries and Taylor-Hypothesis
- Edge states in Blasius boundary layers
- Relation to torque in rotating plane Couette flow

Blasius vs. Suction Boundary Layer

Blasius



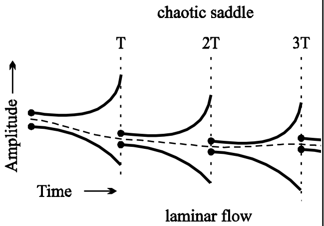
ASBL

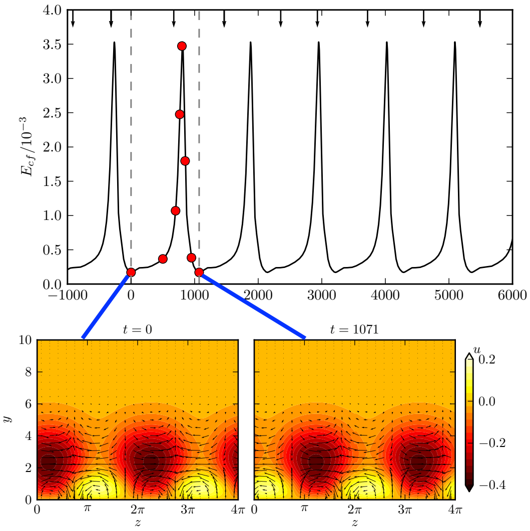


$$\delta(x) = \sqrt{x\nu / U}$$

$$\delta = V_0 / U_0$$

Edge state in ASBL





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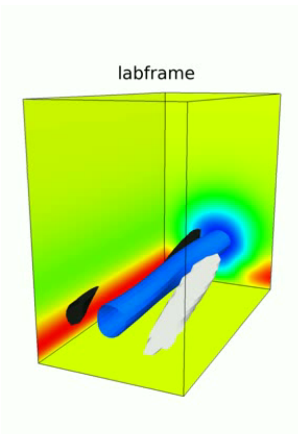
Kreilos, Schneider, Veble, and BE, JFM **726**, 100 (2013)

Vortex dynamics



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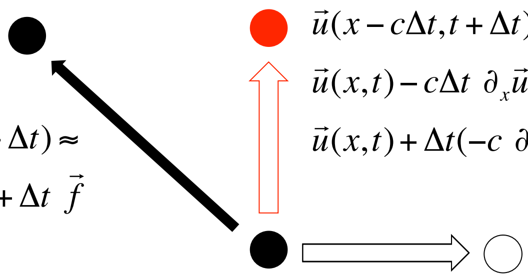
Kreilos and BE, arxiv 1309.4590

Consequences of continuous symmetry (for coherent states):

- Allows for travelling waves: states that move downstream without changing shape
- Allows for relative periodic orbits: states that return up to a translation
- Mixes changes in the flow field and translations thereof

How to remove the translation?

$$\partial_t \vec{u}(x,t) = \vec{f}(\vec{u}(x,t),t)$$



$$\vec{u}(x,t+\Delta t) \approx \vec{u}(x,t) + \Delta t\ \vec{f}$$

$$\vec{u}(x-c\Delta t,t+\Delta t) \approx \vec{u}(x,t) - c\Delta t\ \partial_x \vec{u} + \Delta t\ \vec{f} = \vec{u}(x,t) + \Delta t(-c\ \partial_x \vec{u} + \vec{f})$$

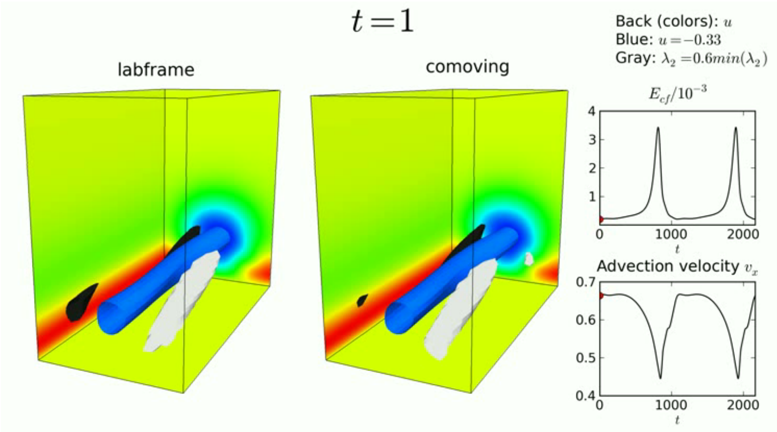
$$\vec{u}(x,t)$$

$$\vec{u}(x-c\Delta t,t) \approx \vec{u}(x,t) - c\Delta t\ \partial_x \vec{u}$$

$$c = \frac{\langle \partial_x \vec{u} \cdot \vec{f} \rangle}{\|\partial_x \vec{u}\|^2}$$

Kreilos and BE, arxiv 1309.4590

Vortex dynamics



Kreilos and BE, arxiv 1309.4590

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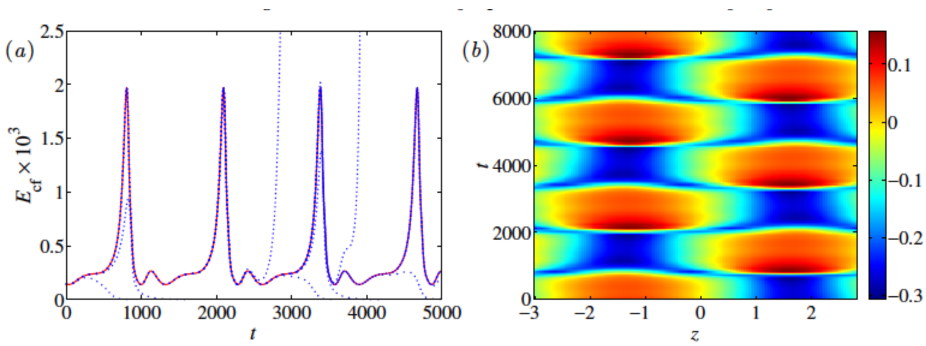
Spatiotemporal dynamics: extended



Khapko, Kreilos, Duguet, Schlatter, Henningson, BE
JFM **717** R6 (2013), arxiv 1308.5531

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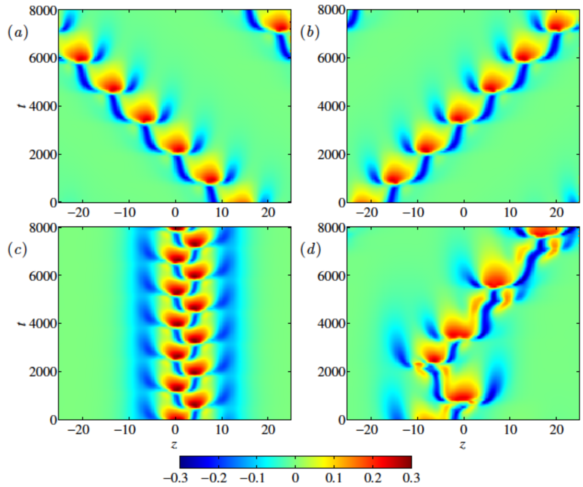
Spatiotemporal dynamics: localized



Khapko, Kreilos, Duguet, Schlatter, Henningson, BE
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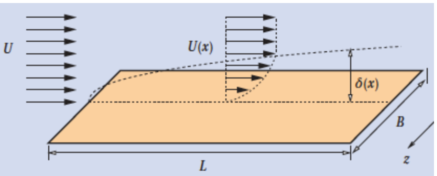
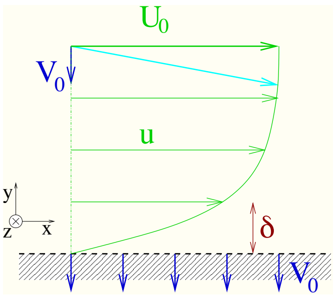
Blasius vs. Suction Boundary Layer

Blasius



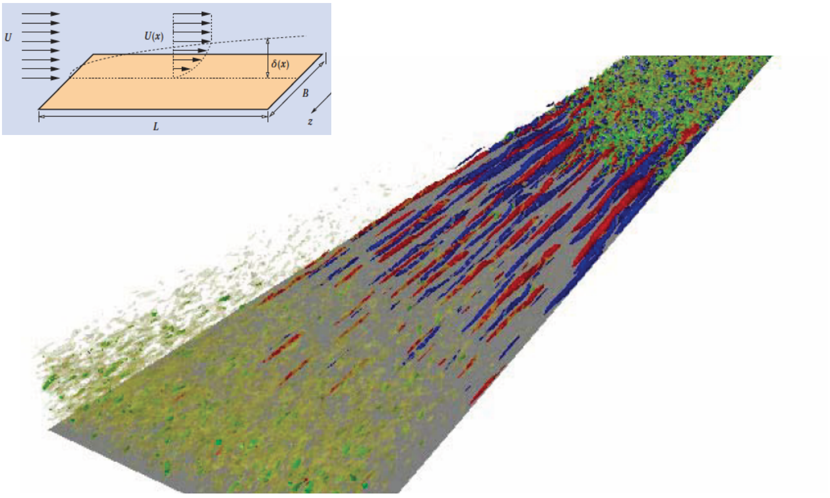
$$\delta(x) = \sqrt{x\nu / U}$$

ASBL



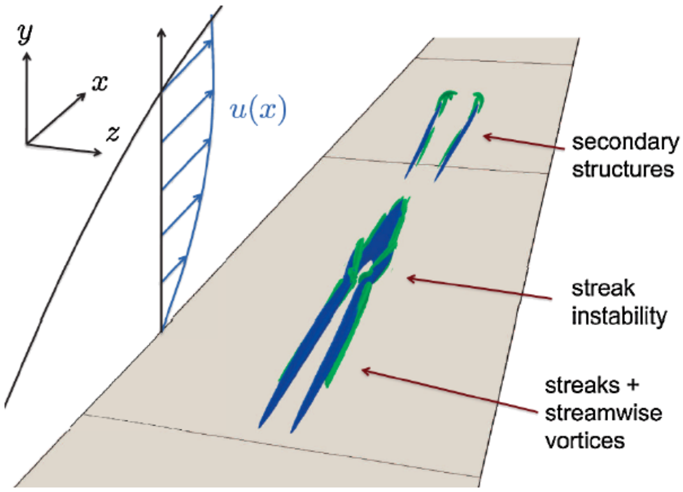
$$\delta = V_0 / U_0$$

Bypass transition in boundary layers

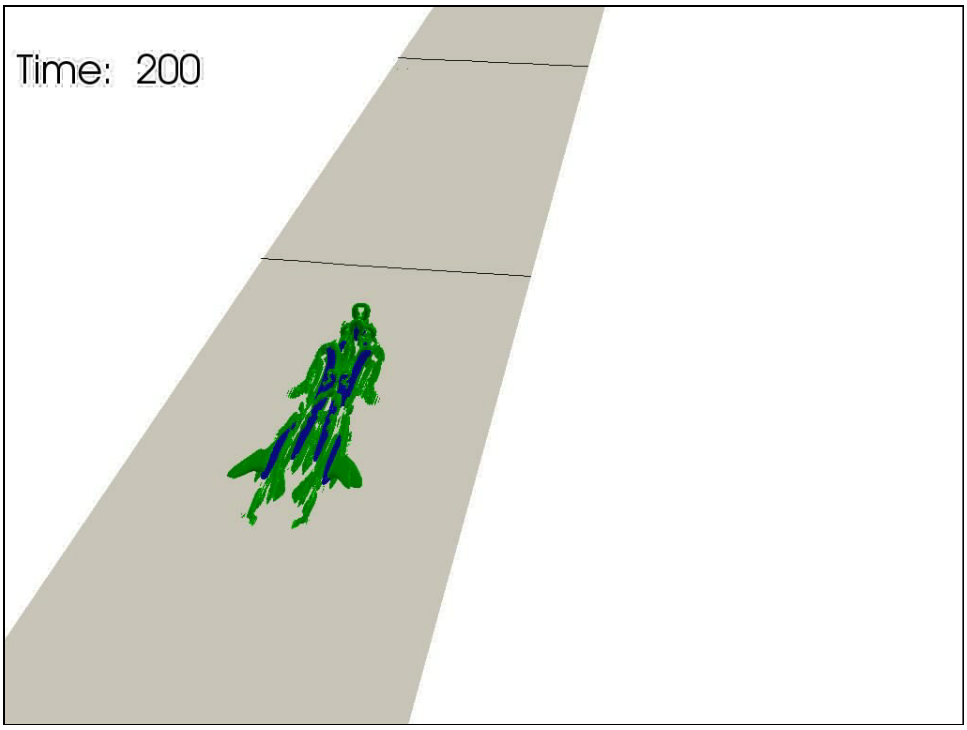


Simulations by P. Schlatter, KTH

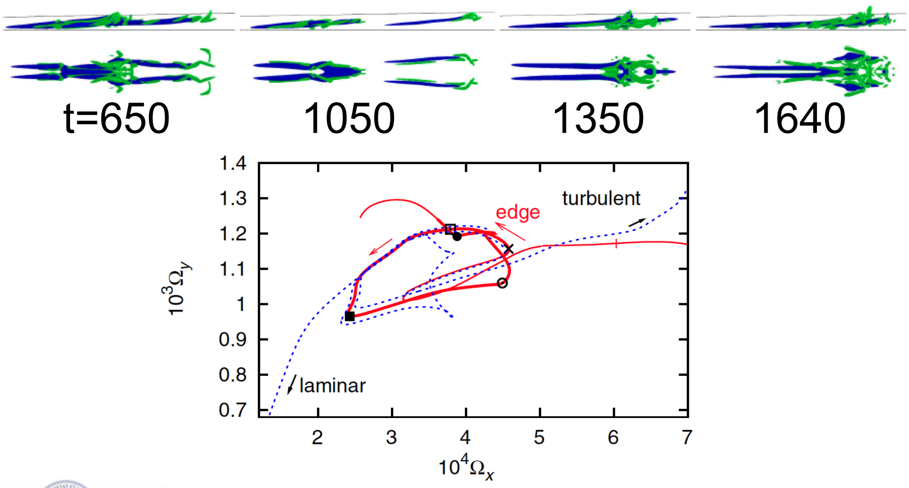
Transition state in Blasius BL



Y. Duguet, P. Schlatter, D.S. Henningson and B.E., PRL **108**, 044501 (2012)



Evolution of the Blasius edge state

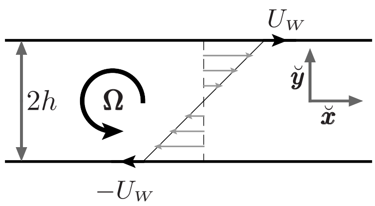


Y. Duguet, P. Schlatter, D.S. Henningson and B.E., PRL **108**, 044501 (2012)

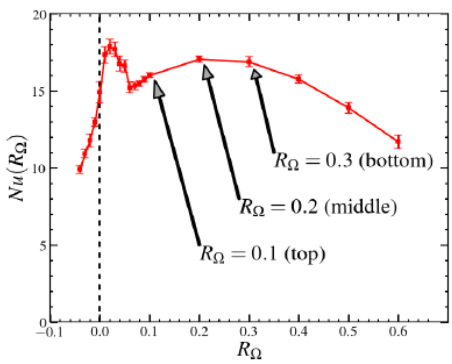
Rotating plane Couette flow

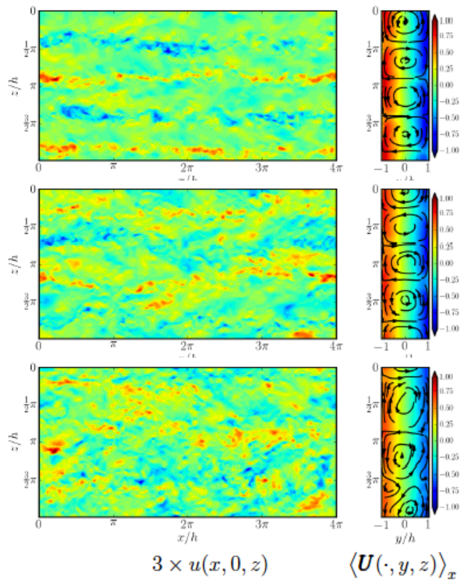
$$\mathrm{Re} = U_W h / \upsilon$$

$$R_\Omega = 2\Omega h / U_W$$



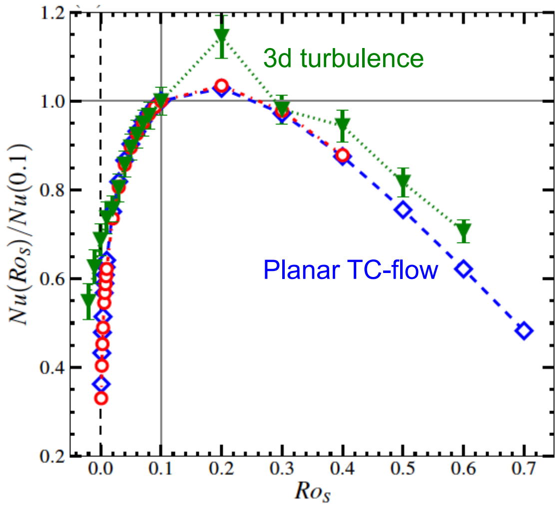
Rotating Taylor-Couette flow



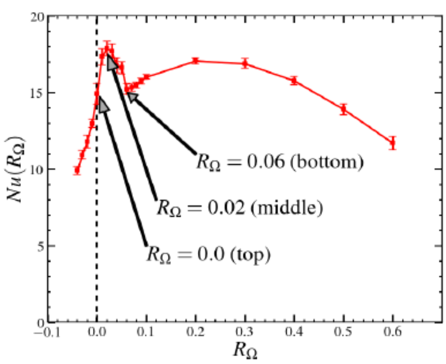


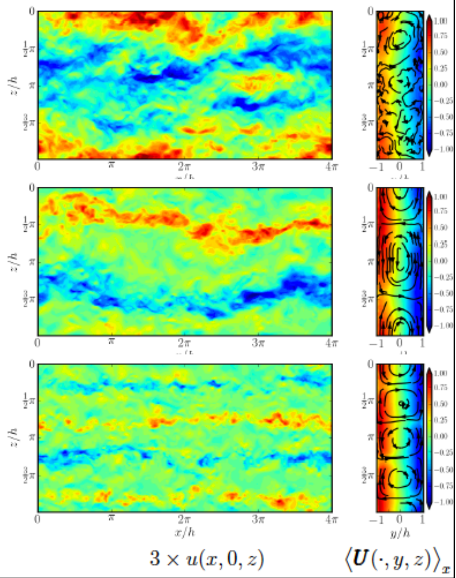
$$\mathrm{Re} = 20400$$

Rotating plane Couette flow



Rotating Taylor-Couette





Re =20400

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Thanks to:

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