Reynolds Number Scaling of Dissipative Motions

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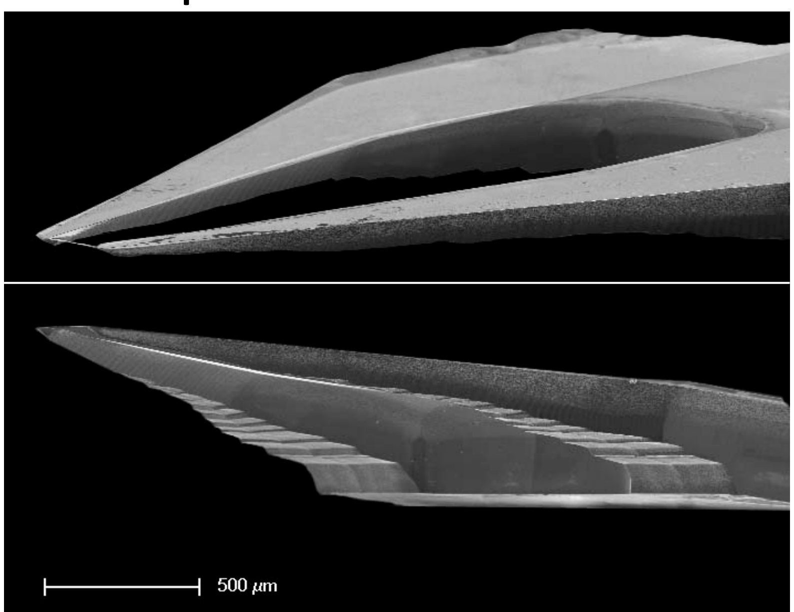
Motivation

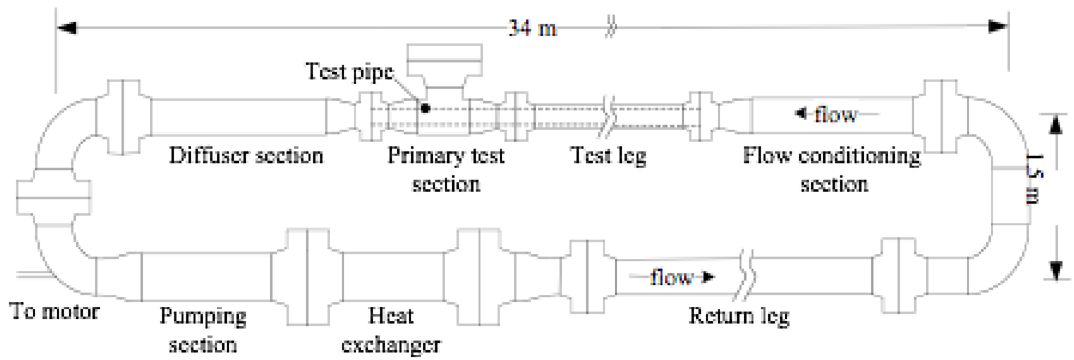
- Recent instrumentation advances in the form of the nano-scale thermal anemometry probe (NSTAP) developed at Princeton University has allowed the measurement of highly resolved turbulence in high Reynolds number canonical wall-bounded flows
- Allows the examination of the Reynolds number dependence dissipation scales of turbulence



NSTAP:

- Platinum wire
- Sensing volume of 100nm x 2 μm x 60 μm
- Operated with a conventional anemometer box (CTA)
- Frequency response >150kHz

Pipe Flow Measurements

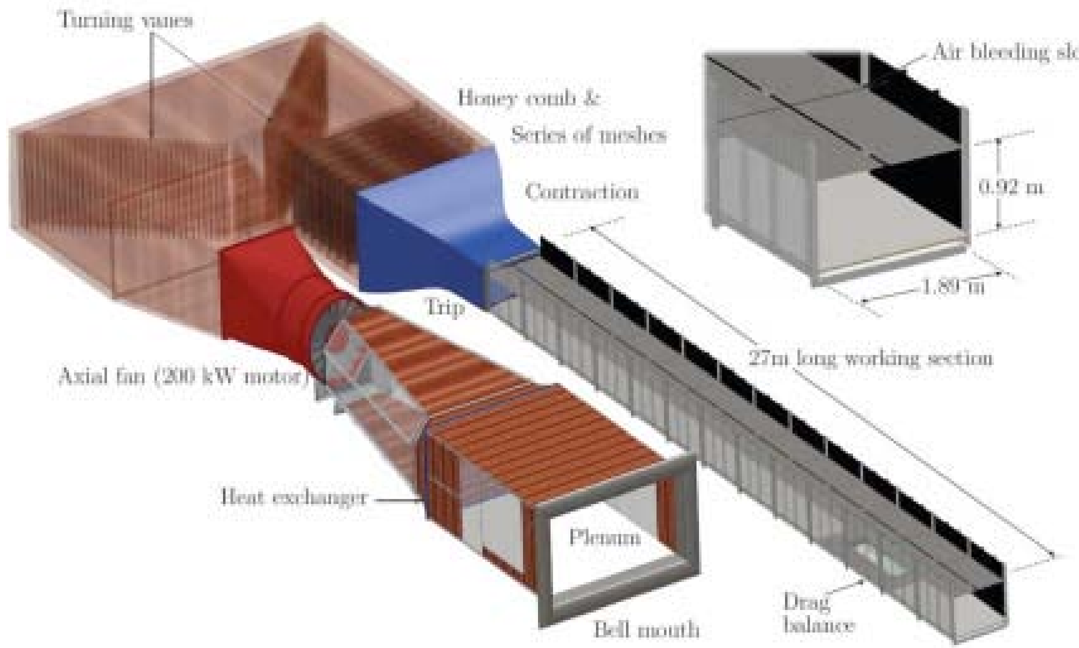


- Pipe flow experiments performed in Princeton Superpipe
  - Small scales resolved for $2000 < Re_\tau < 10{,}000$
- Frequency response>140 kHz
  - Sample frequency 300 kHz
  - $\Delta t < 0.05\,\tau_\eta$ for $2000 < Re_\tau < 10{,}000$

Re_D	Re_τ	ν/u_τ (μm)	l^+	l/η
81,000	1985	33	1.8	<0.8
146,000	3334	19	3.1	<1.4
247,000	5412	12	5	<2.3
512,000	10,481	6.2	9.7	<4.4
1,100,000	20,250	3.2	18.8	<8.5
2,100,000	37,960	1.7	35.0	<16
4,000,000	68,160	0.95	63.2/31.7	<29/14
6,000,000	98,190	0.66	45.5	<20.7

Turbulent Boundary Layer Measurements

- Zero pressure gradient boundary layer experiments performed in University of Melbourne HRNBLWT
- $Re_\tau = 7{,}500$
  - $\nu/u_\tau \sim 27\ \mu m$
  - $l^+ \sim 2$
  - $l/\eta < 1$
- $Re_\tau = 12{,}000$
  - $\nu/u_\tau \sim 22\ \mu m$
  - $l^+ \sim 2.7$
  - $l/\eta < 1$
- Frequency response>140 kHz
  - Sample frequency 250 kHz
  - $\Delta t < 0.02\,\tau_\eta$ for $Re_\tau = 7{,}500$
  - $\Delta t < 0.03\,\tau_\eta$ for $Re_\tau = 12{,}000$



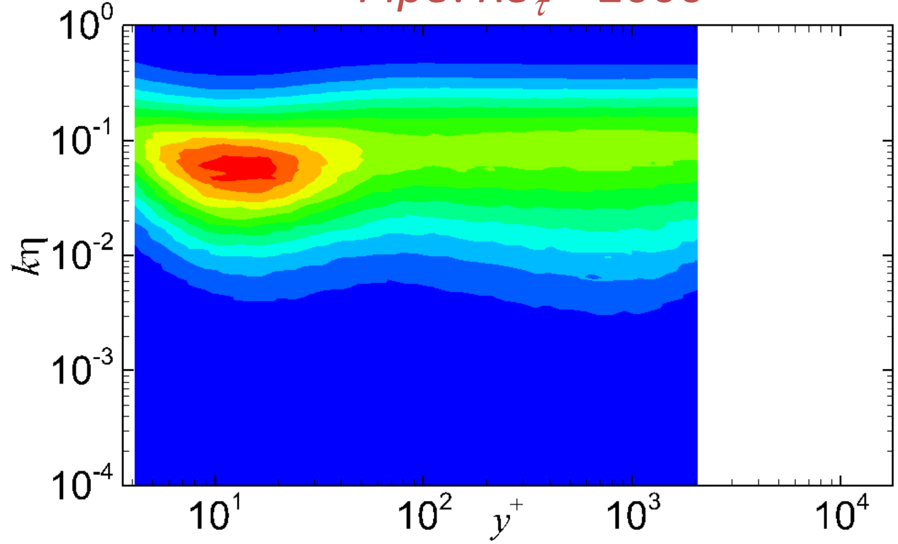
http://www.mech.unimelb.edu.au/fluid-mechanics/walter-bassett.html

Estimated Dissipation Spectra

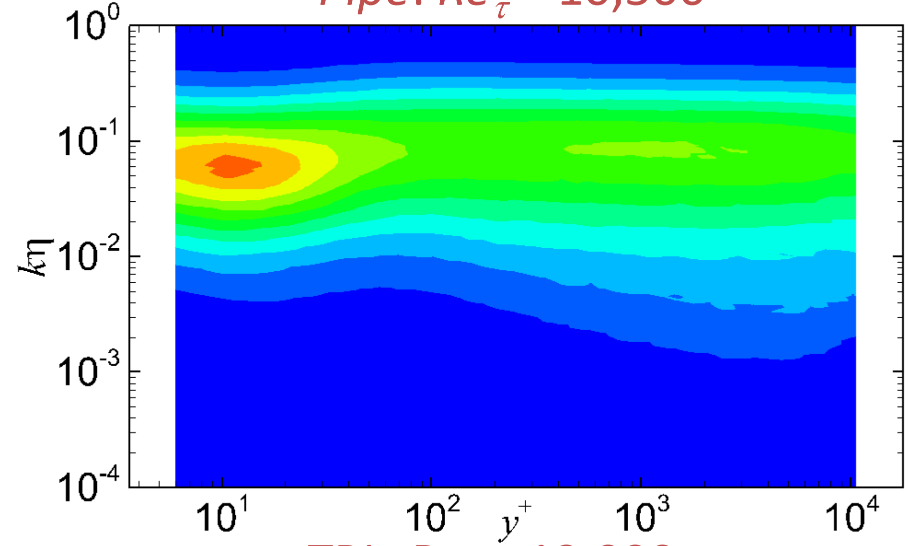
$$D(k,y) \approx 15\nu k^2 \Phi(k,y)$$

$$\varepsilon \approx \int_0^\infty 15\nu k^2 \Phi(k,y)\,dk$$

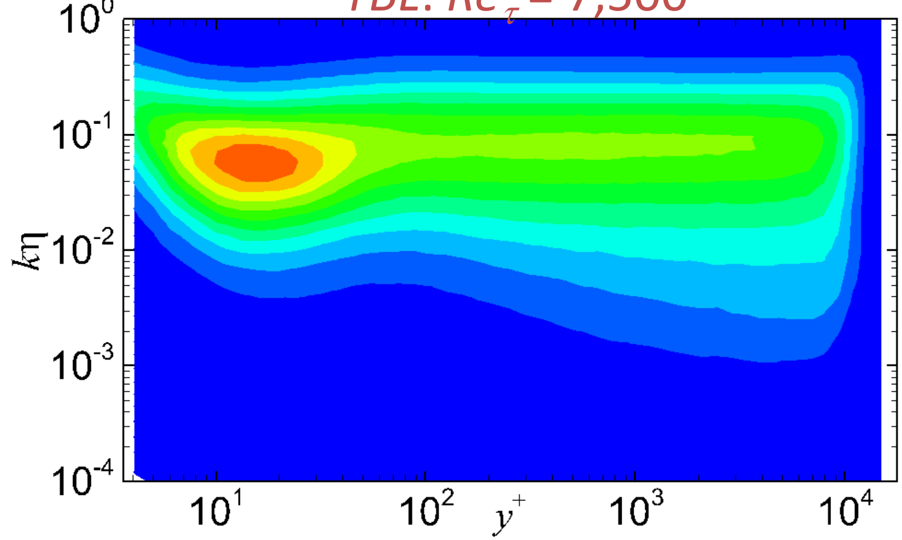
Pipe: Re_τ = 2000



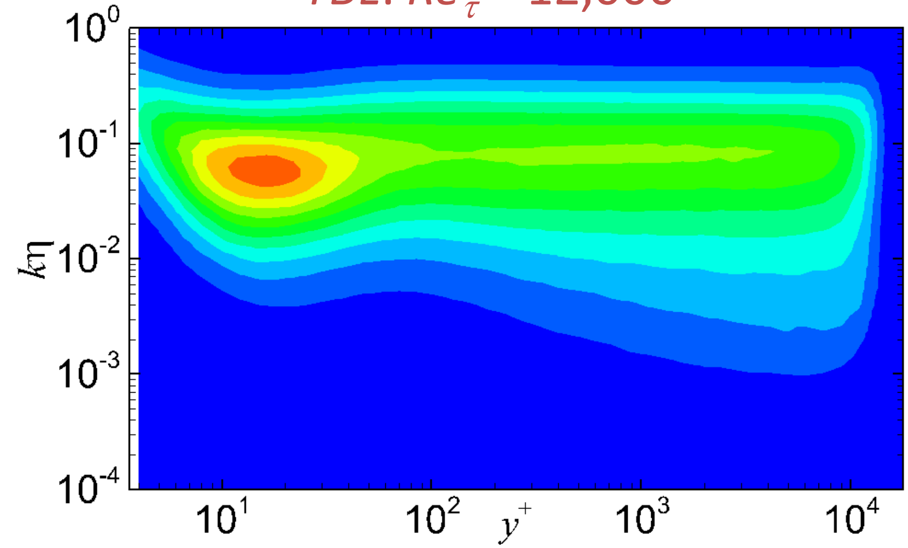
Pipe: Re_τ = 10,500



TBL: Re_τ = 7,500

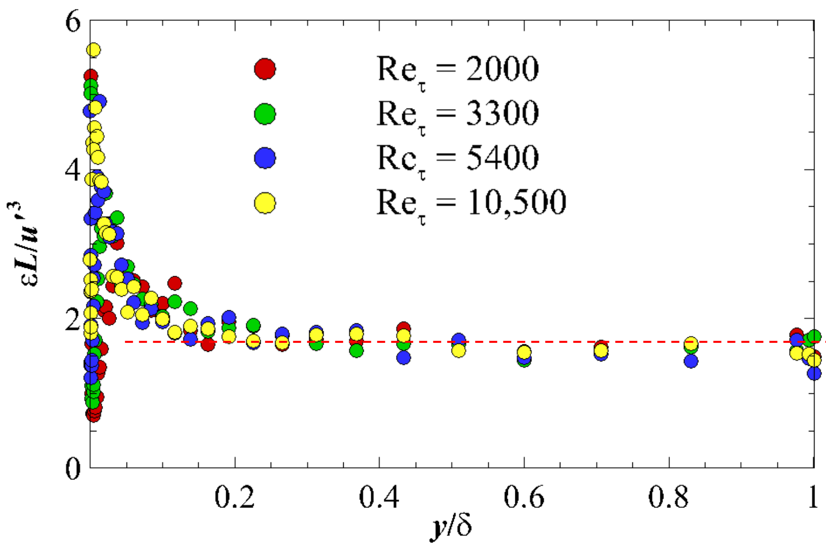


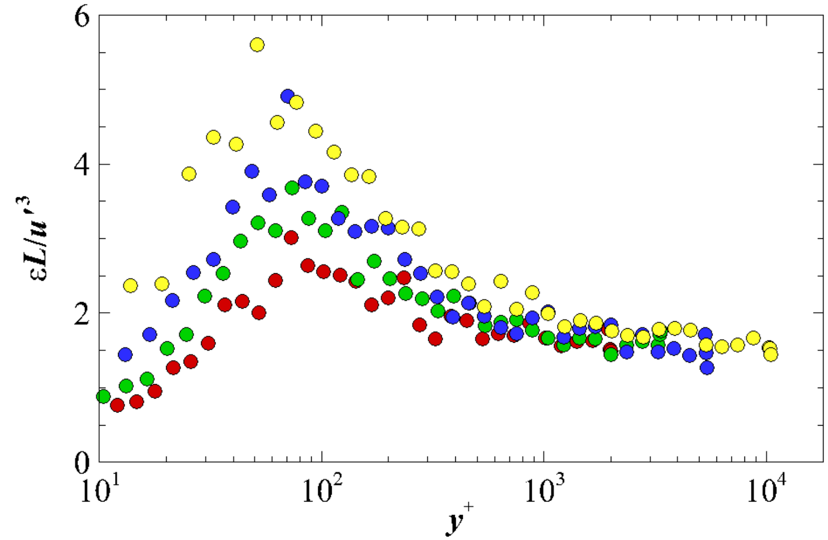
TBL: Re_τ = 12,000



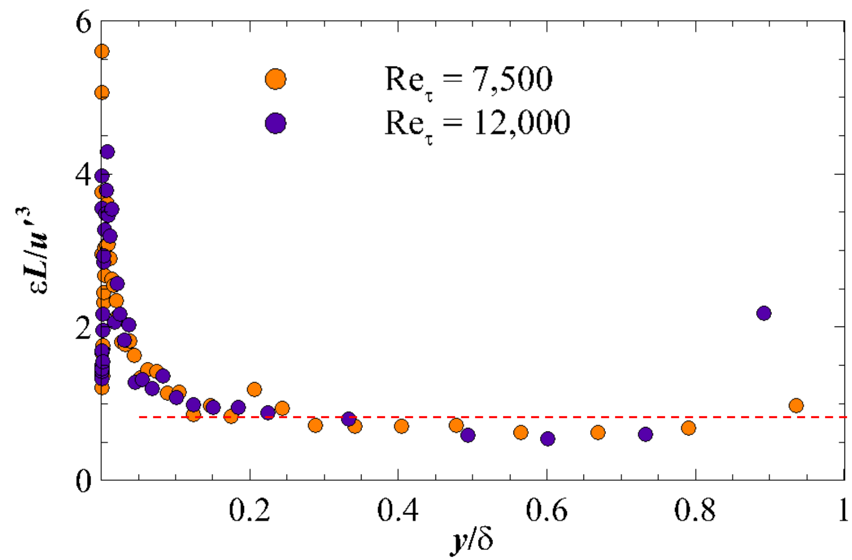
Scaled Dissipation Rate

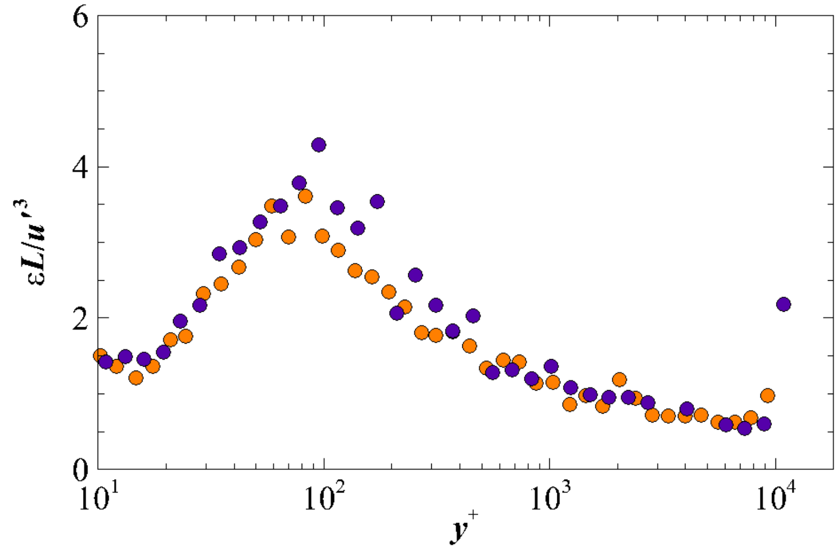
Pipe





TBL

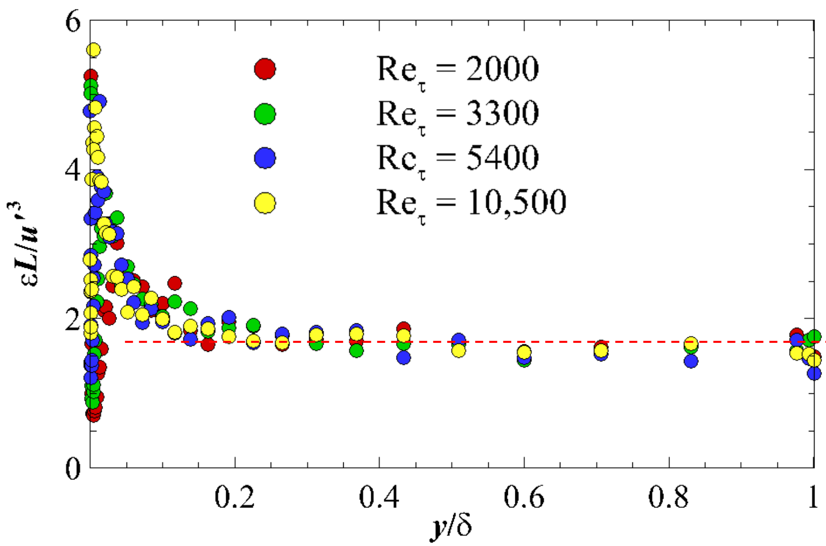


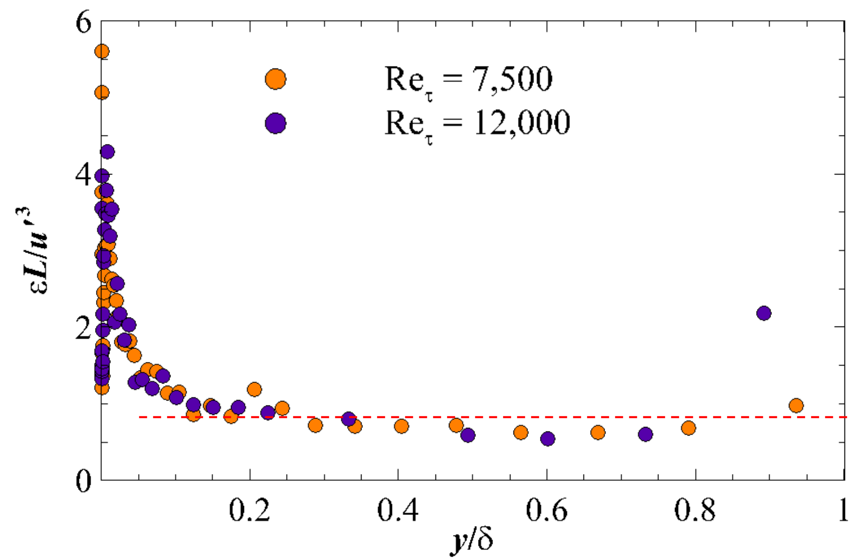


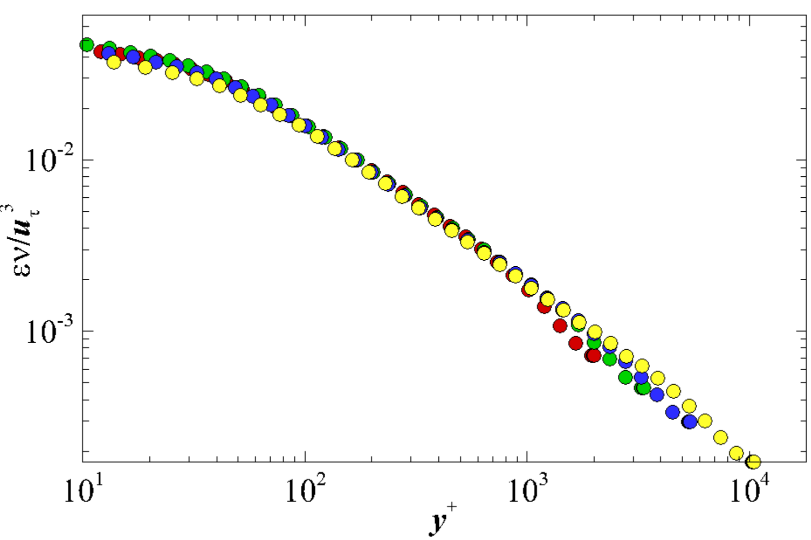
Scaled Dissipation Rate

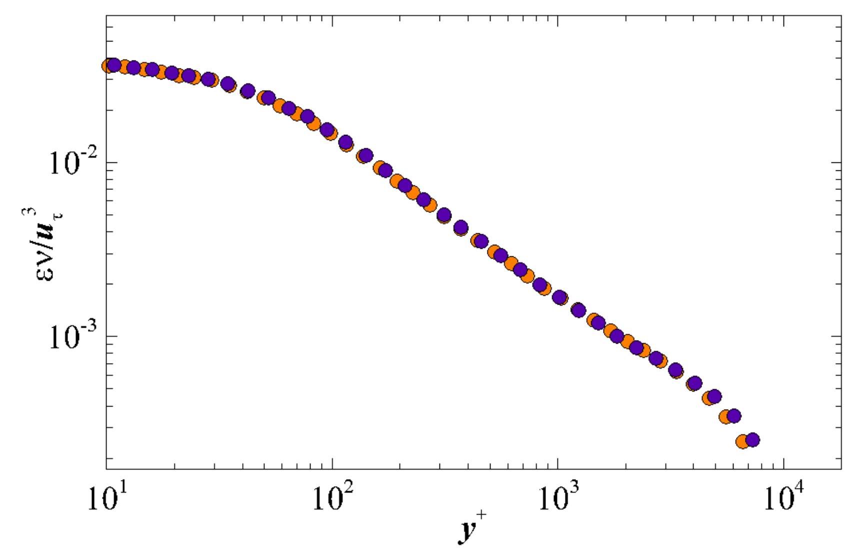
Pipe

TBL



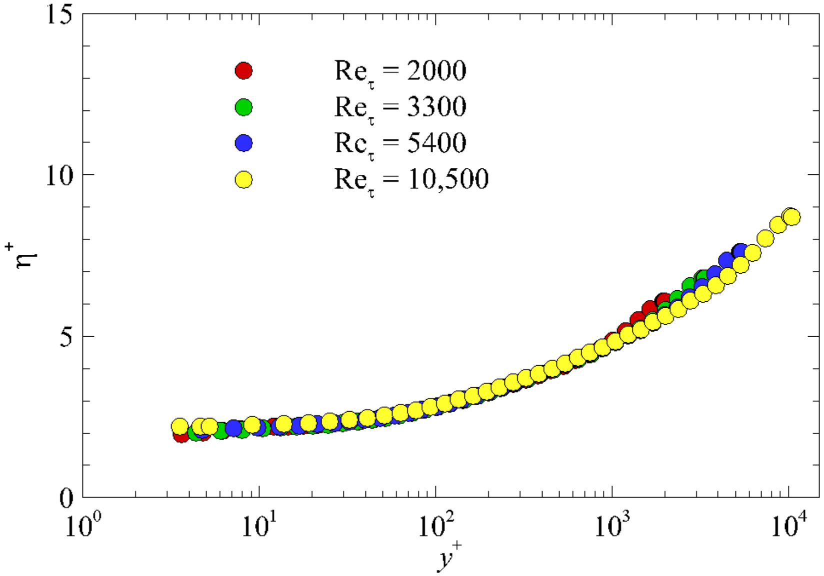


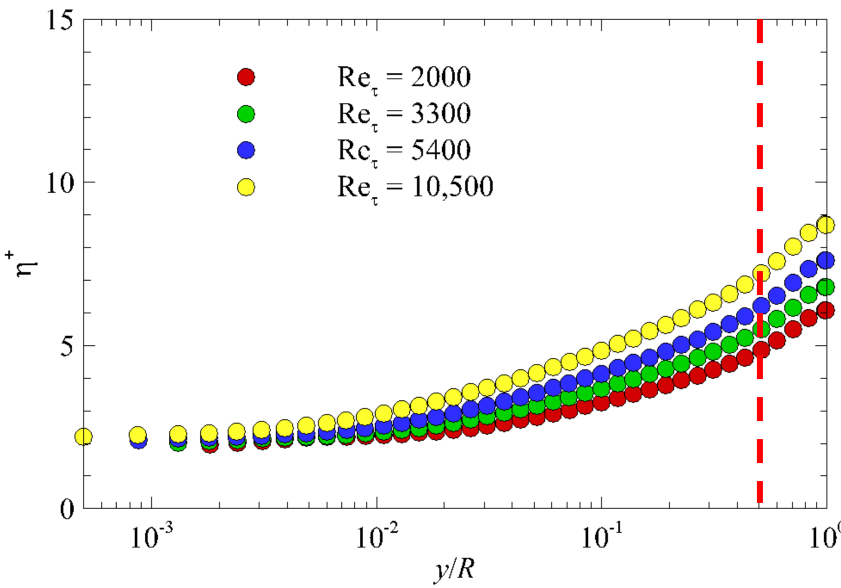




Kolmogorov Scales

Pipe





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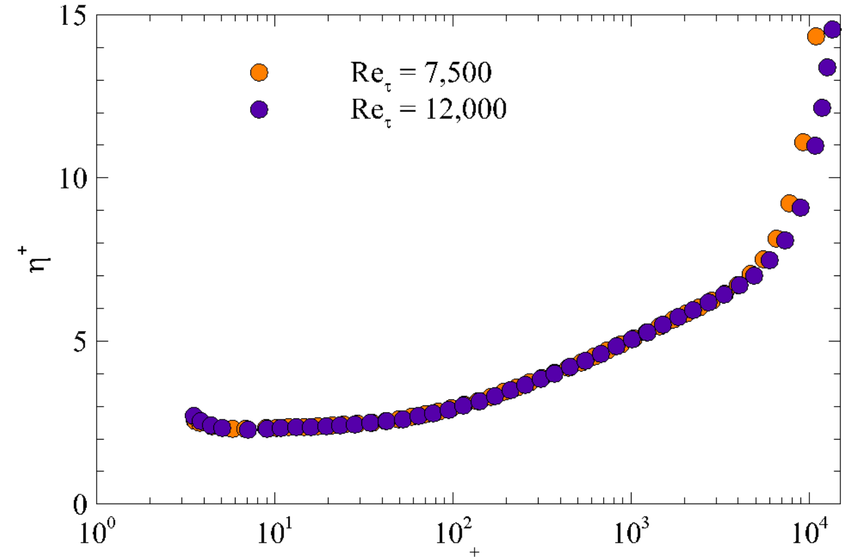
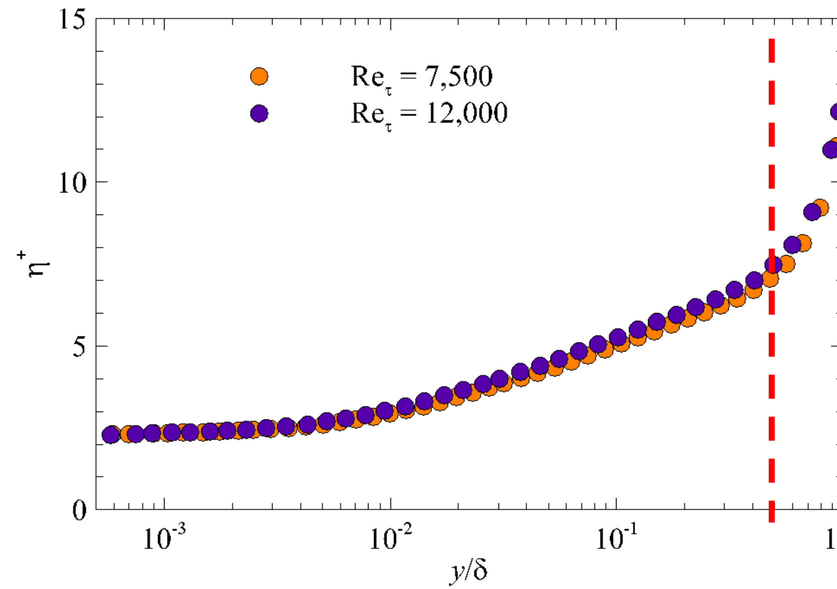




Dissipation Scale Distribution

- Classically:
  - Dissipation occurs around Kolmogorov scales:

$$\eta_K \equiv \frac{\nu^{3/4}}{\langle \varepsilon \rangle^{1/4}} \qquad u_\eta \equiv \langle \varepsilon \rangle^{1/4} \nu^{1/4} \qquad \tau_\eta \equiv \frac{\nu^{1/2}}{\langle \varepsilon \rangle^{1/2}}$$

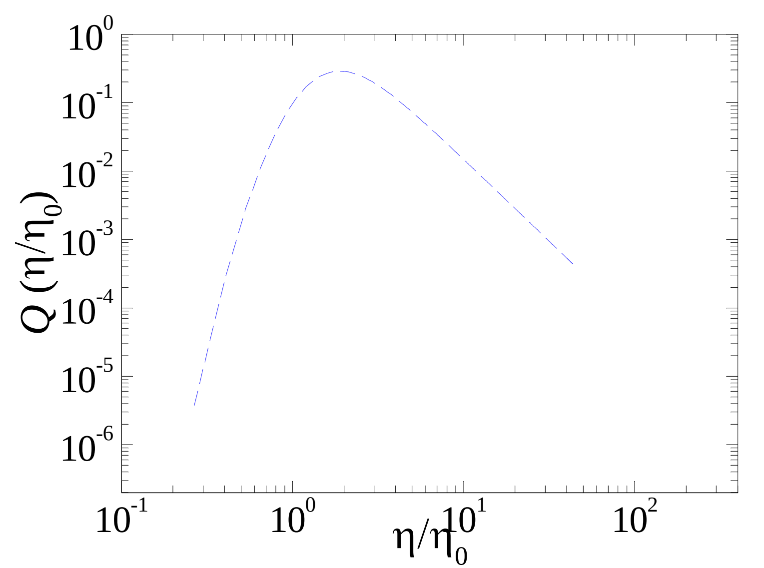
- Yakhot (2005, 2006):
  - Dissipation scale as a fluctuating random field

$$\delta u_\eta = |u(x+\eta) - u(x)| \qquad \frac{\delta u_\eta \eta}{\nu} = \mathrm{Re}_\eta = O(1)$$

$$Q(\eta) = \left|\frac{\mathrm{d}u_\eta}{\mathrm{d}\eta}\right| P\left(u_\eta \middle| \frac{u_\eta \eta}{\nu} = c\right) = \frac{c\nu}{\eta^2} P\left(u_\eta \middle| \frac{u_\eta \eta}{\nu} = c\right)$$

$$Q(\eta) = \frac{1}{\pi\eta\sqrt{b\log(L/\eta)}} \int_{-\infty}^{+\infty} \mathrm{d}x \times \exp\left[-x^2 - \frac{\left(\log\left(\frac{\sqrt{2}xR}{c}\left(\frac{\eta}{L}\right)^{a+1}\right)\right)^2}{4b\log(L/\eta)}\right]$$

(constructed from the velocity increments by using scaling arguments and the Mellin transform)



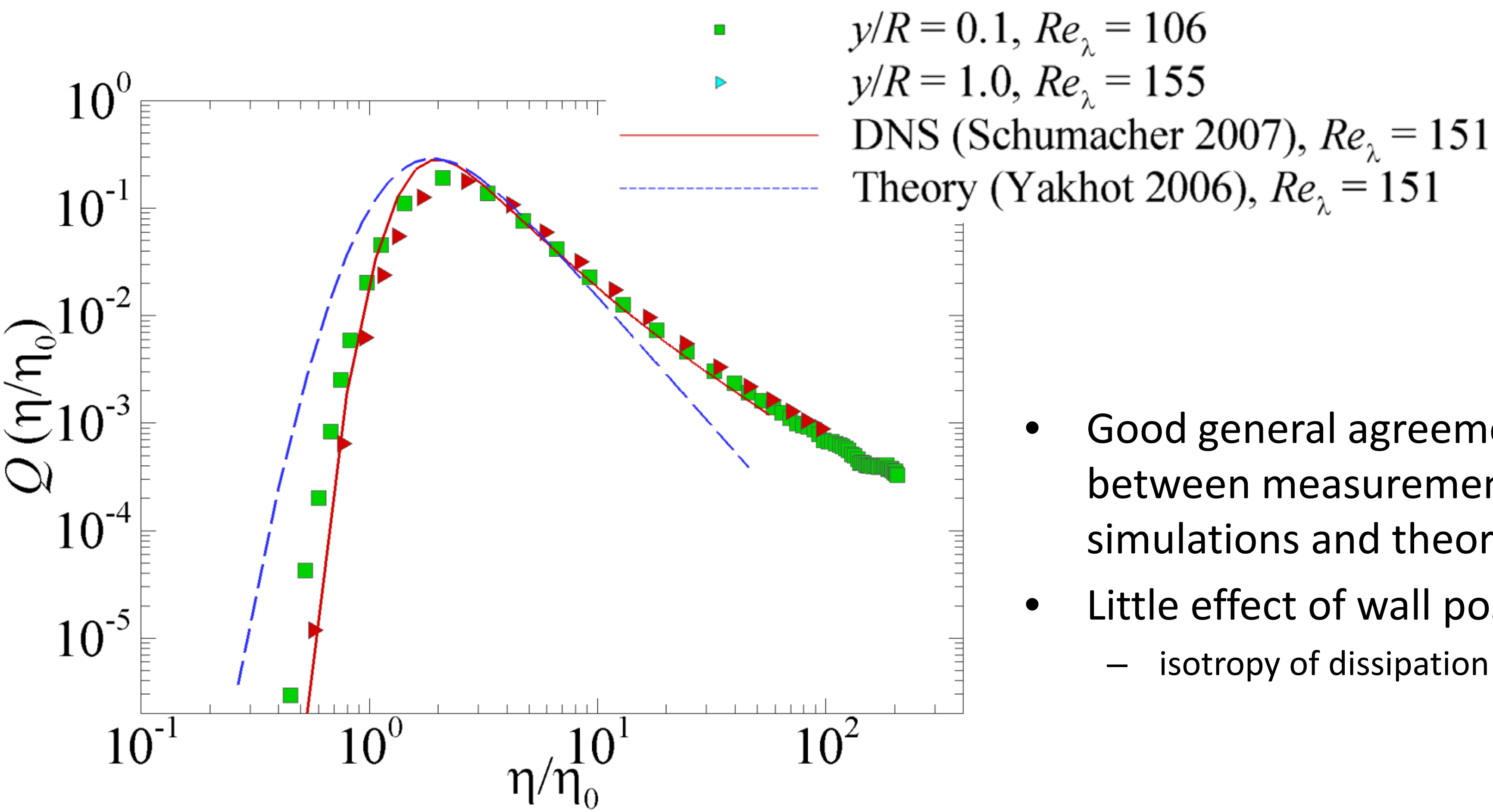
Background

- Experimental measurement of probability density function (pdf) of dissipation scales:
  - Only streamwise direction available through application of Taylor's hypothesis
  - Dissipation scale when

$$\frac{\delta_\eta u \eta}{\nu} \approx \frac{(U(t+\eta/\overline{U}) - U(t))\eta}{\nu} \approx 1$$

  - Count occurrences where 0.9 < $\delta_\eta u \eta / \nu$ < 2 over the range 0 < η < L.

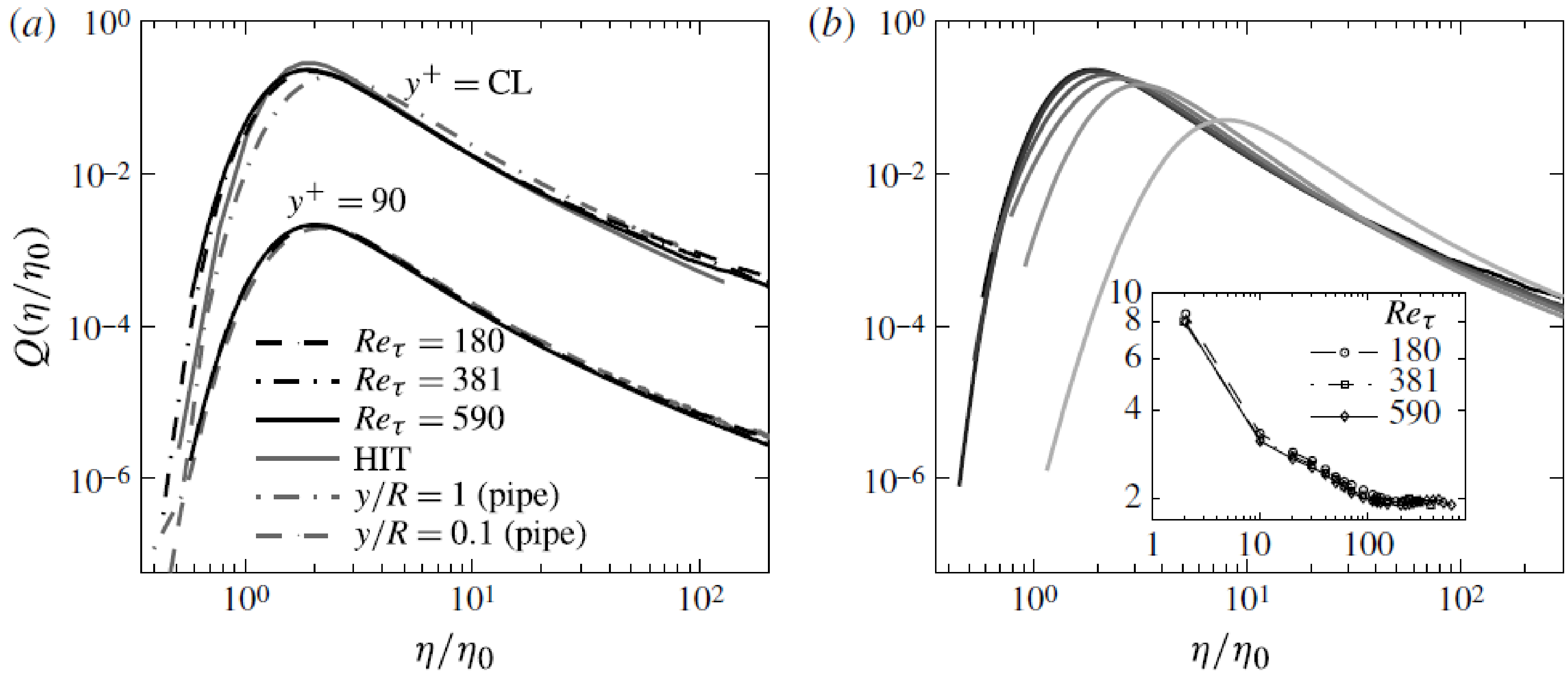
Measured Dissipation Scales



- Good general agreement between measurements, simulations and theory
- Little effect of wall position
  - isotropy of dissipation scales

Bailey, Hultmark, Schumacher, Yakhot, Smits (PRL) 2009

Dissipation Scale Distribution



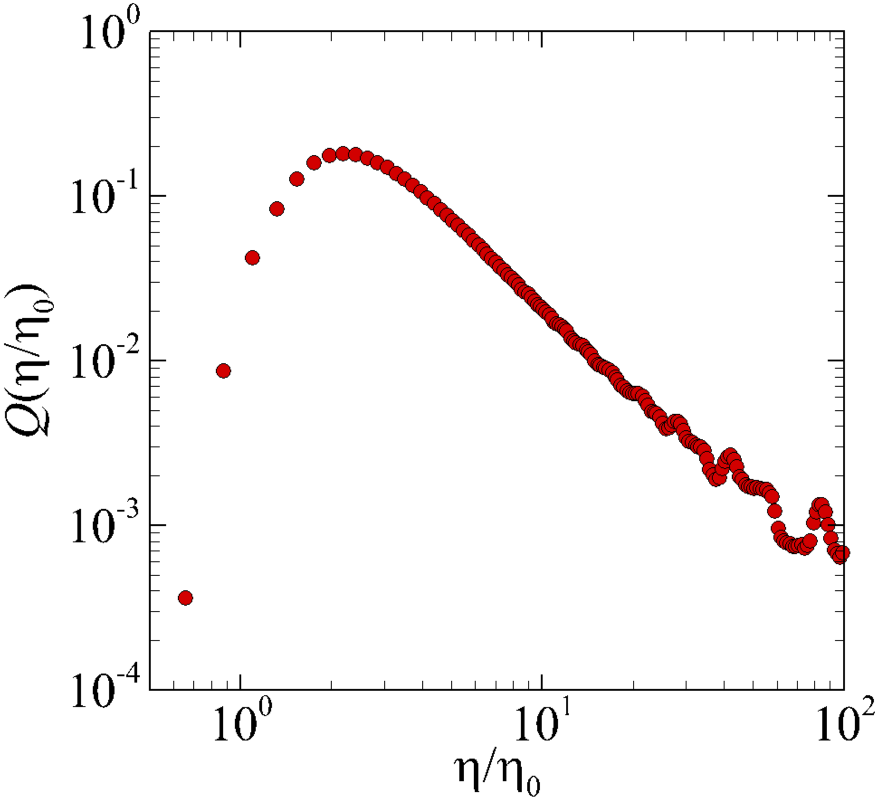
Hamlington, Krasnov, Boeck, Schumacher (JFM) 2012

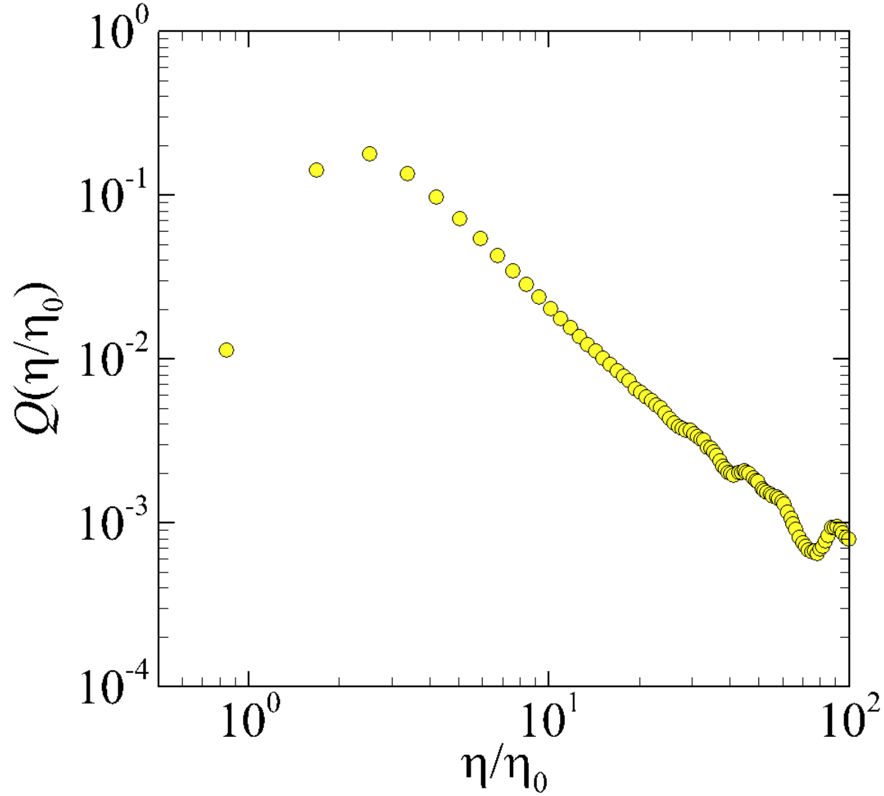
- High spatial resolution DNS
  - PDFs of dissipation scale closely agree with homogeneous results for $y^+ > 100$
  - Suggest that there is a universal, flow-independent behavior of the dissipation scales

Measured Dissipation Scale Distributions

Pipe: Re_τ = 2000 | *Pipe:* Re_τ = 10,500

y^+ = 1000



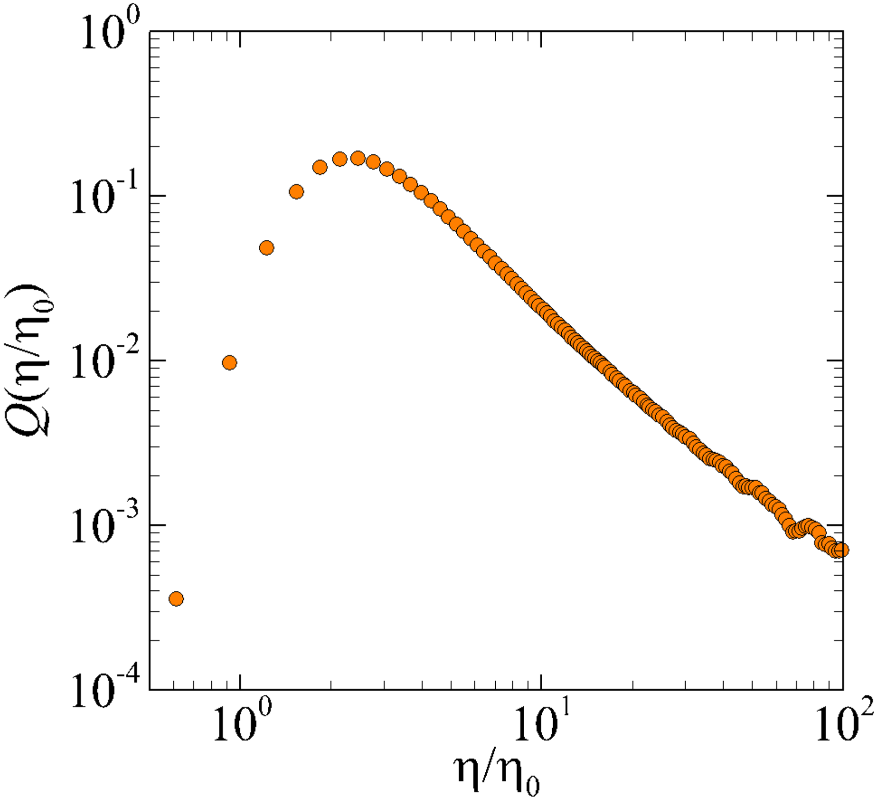


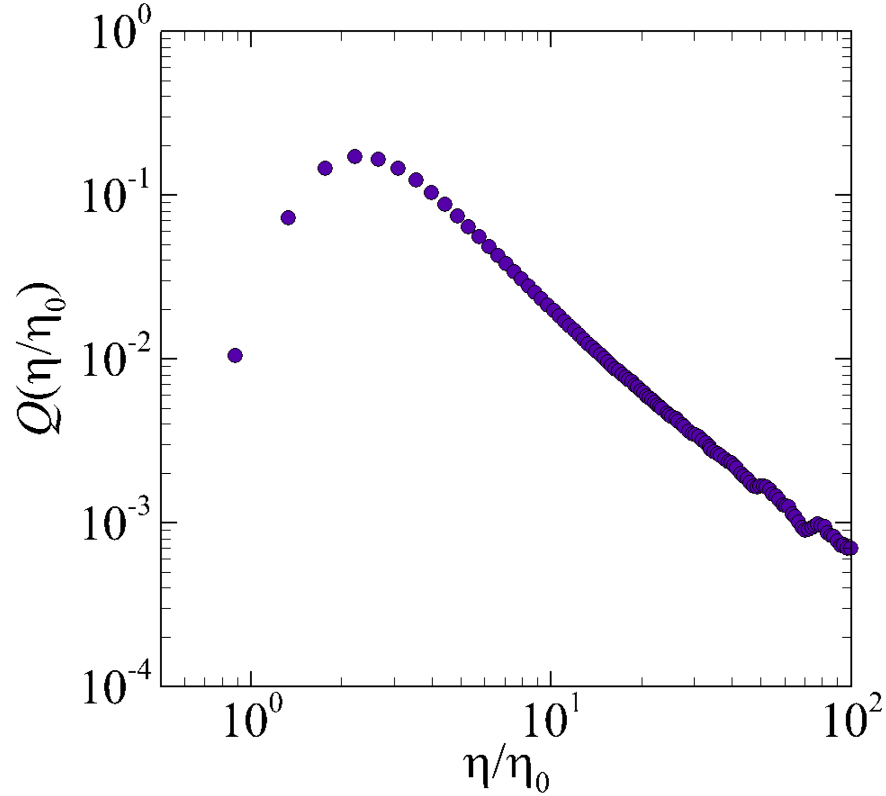
- Structure of PDF qualitatively the same as for lower Reynolds number pipe and homogeneous isotropic turbulence

Measured Dissipation Scale Distributions

TBL: $Re_\tau = 7500$ | *TBL:* $Re_\tau = 12000$

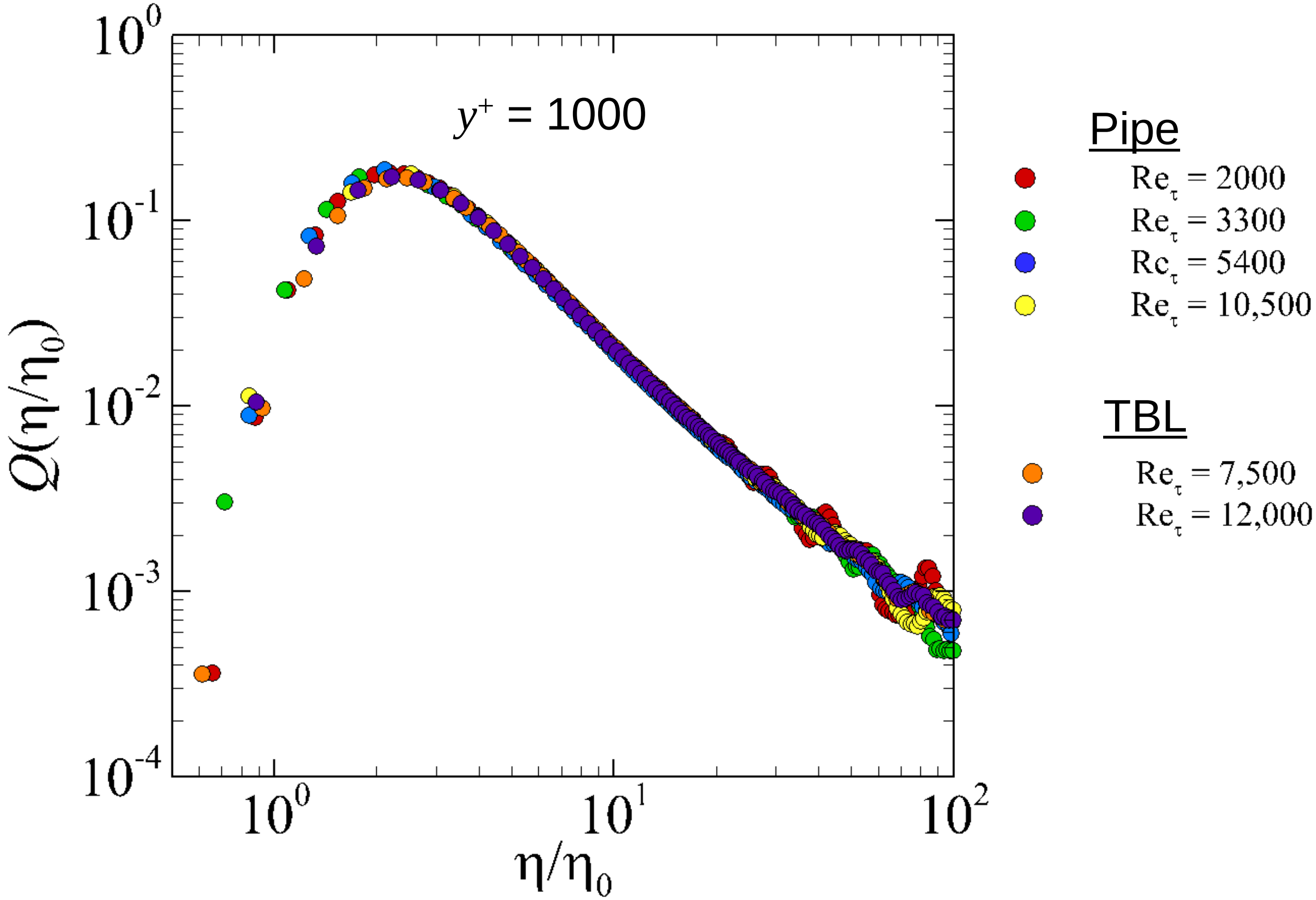
$y^+ = 1000$





- Structure of PDF qualitatively the same as for lower Reynolds number pipe and homogeneous isotropic turbulence

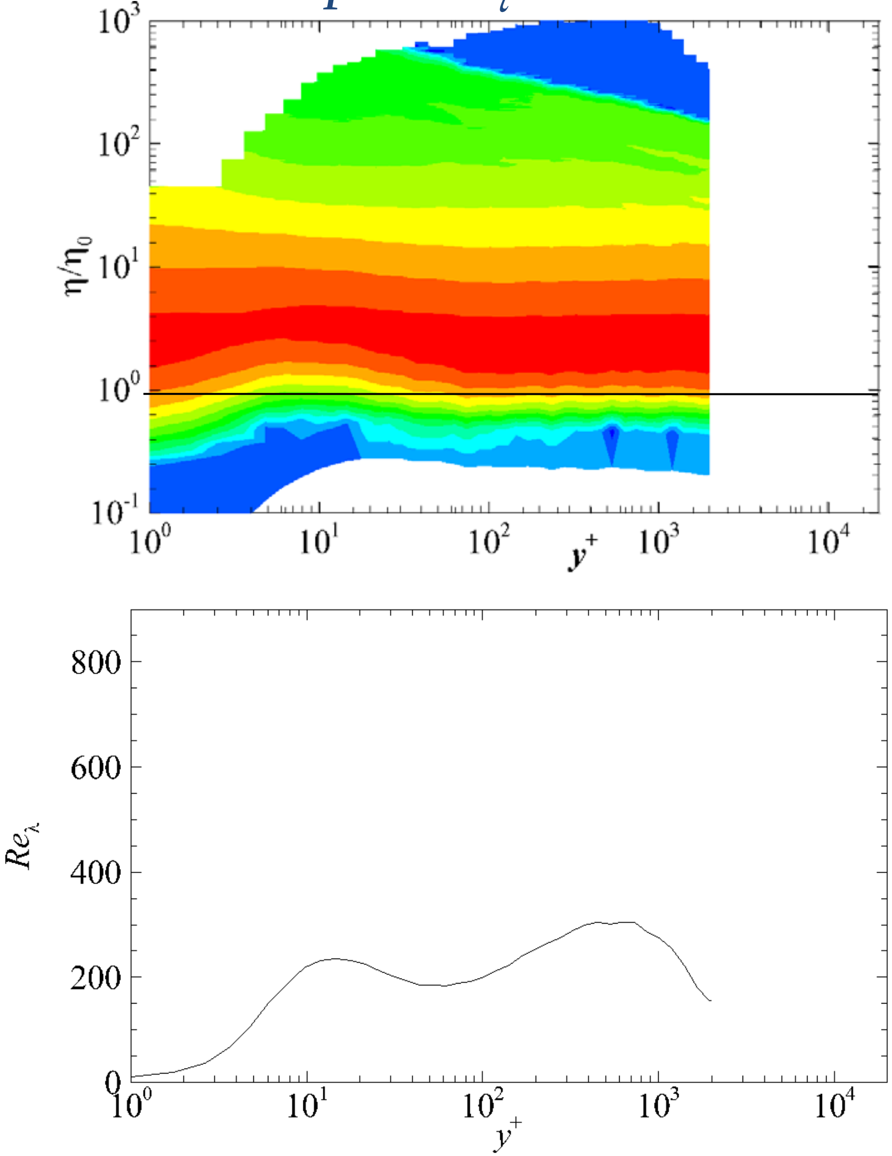
Measured Dissipation Scale Distributions



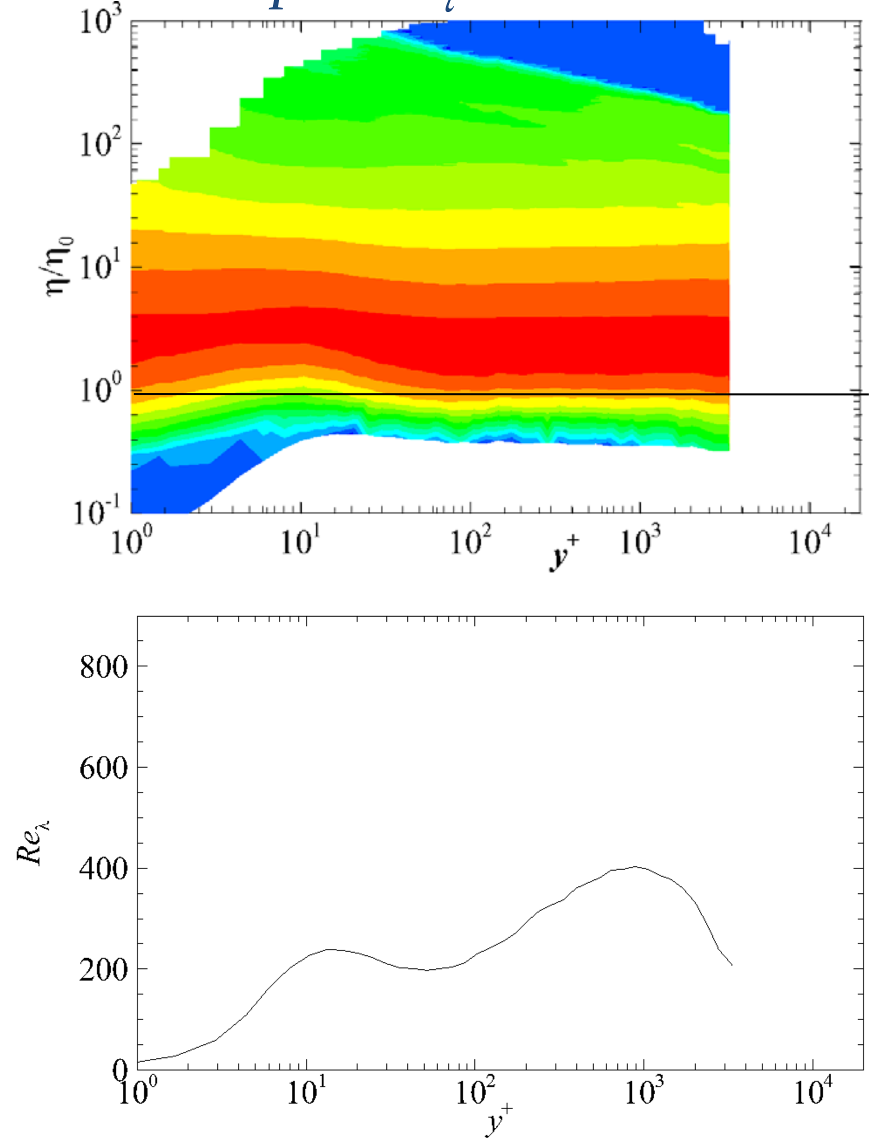
- Structure of PDF virtually identical for both pipe and turbulent boundary layer from 2000 < Re_τ < 12,000

Measured Dissipation Scales

Pipe: Re_τ = 2000

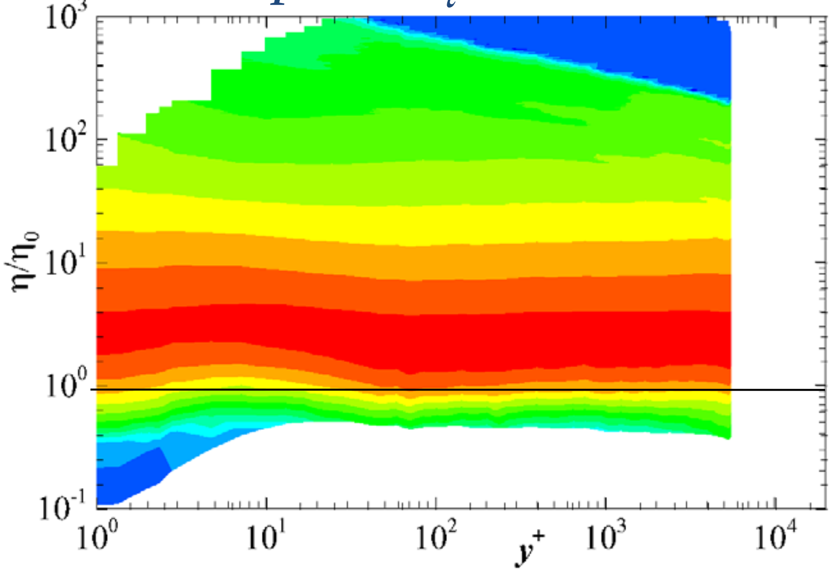


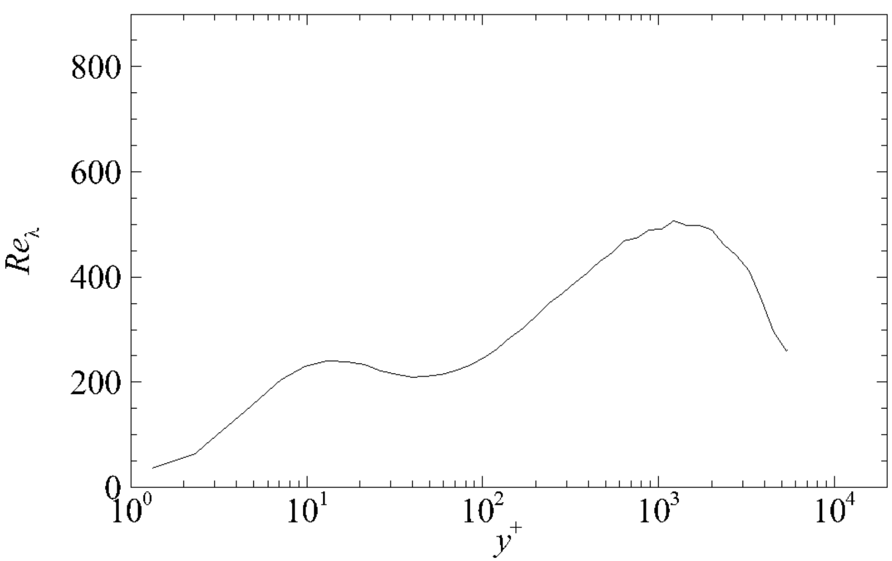
Pipe: Re_τ = 3300



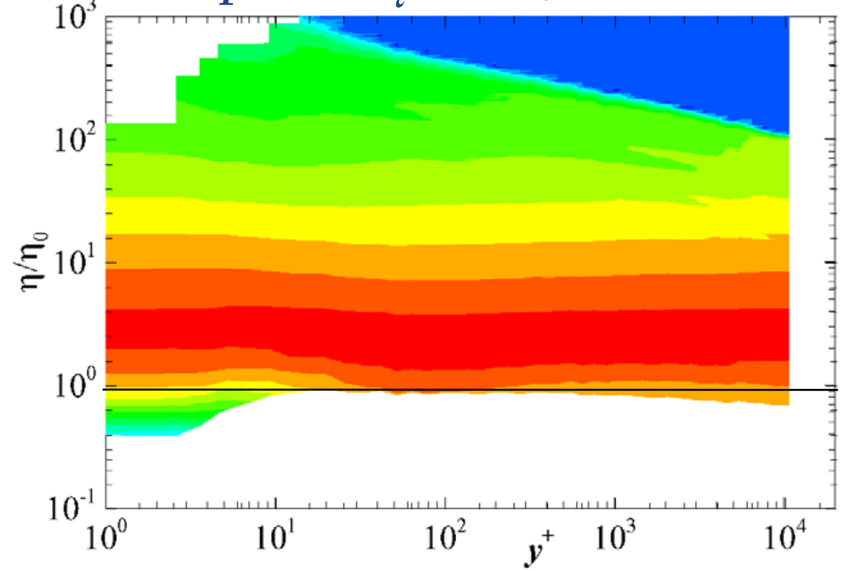
Measured Dissipation Scales

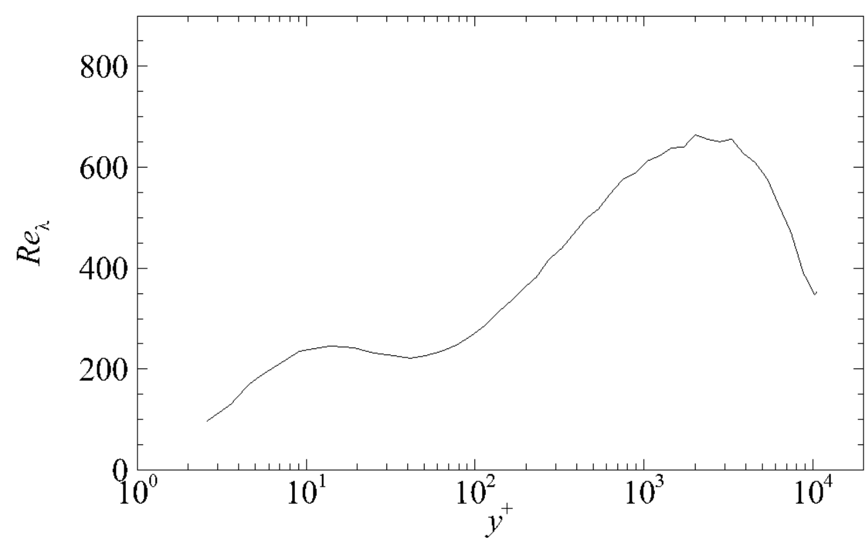
Pipe: Re_τ = 5400



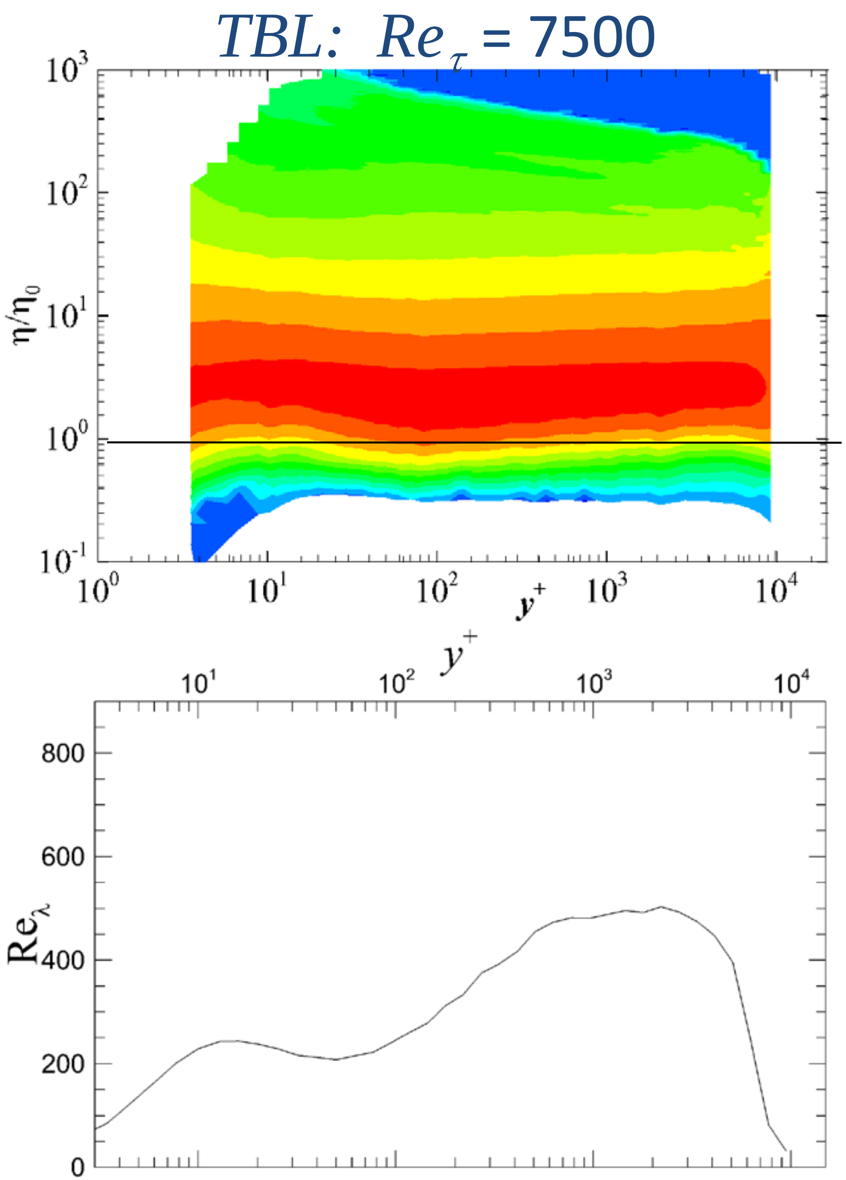


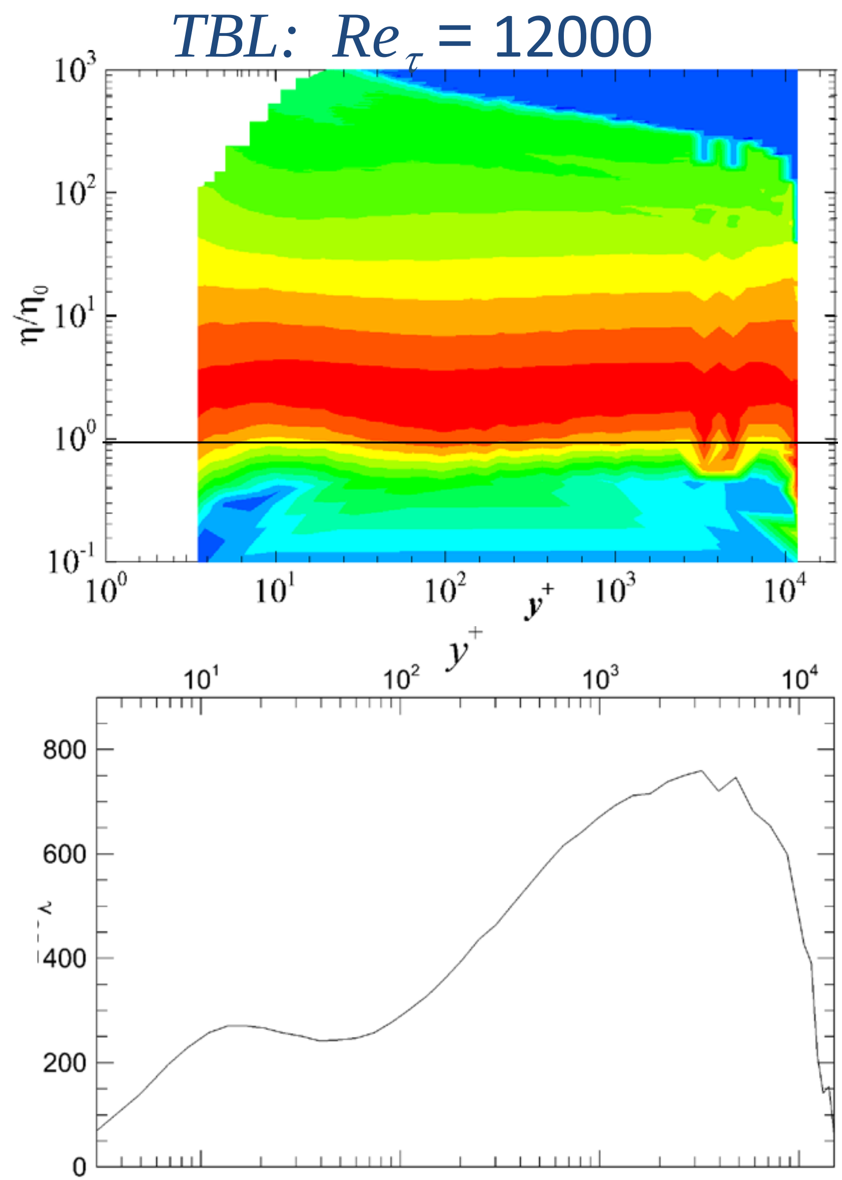
Pipe: Re_τ = 10,500





Measured Dissipation Scales





Conclusions

- Highly resolved measurements of the dissipation scales in high Re lab flows were made in pipe and turbulent boundary layers over a wide range of Reynolds numbers
- Good general agreement in PDFs of dissipation scales between experiment, simulation and theory
- Near universality of the PDFs for $y^+>100$
  - At most, there is weak wall-normal/Re_λ dependence observed in wall flows at high Re_τ for $y^+>100$